AN APPROACH TO THE DYNAMIC CLIMATOLOGY OF NEW ZEALAND

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ABSTRACT

The explanatory description of the climate of the New Zealand region is approached from a statistical time scale analysis of meteorological data for the year August 1962 to August 1963. The equations for the calculation of the variance spectra for individual series and of the interrelationships spectra for two series are given and their application to meteorological data discussed.

Calculations of the variances reveal that the spectra of meteorological elements are continuous but that some frequencies appear to stand out more than others. The periods of one day and one year are dominant, especially in temperature, but also frequencies of about 0.0250, 0.0625, 0.1500 and 0.2000 cycles per day appear consistently above the others on a linear frequency scale.

The frequencies of 0.0750 and 0.1375 cycles per day have been selected for detailed study of the interrelationships. They show that there is moderate coherence between data at these scales, and that models constructed from the calculations show a high similarity to those of the spatial domain. Together with the mean values their role in the maintenance of the general circulation is outlined. A direct and objective relationship between the general circulation and the large scale features of climate is thus evolved.

CHAPTER 1

INTRODUCTION

1.1 Aims

Dynamic climatology is here defined as "the explanatory description of climate in terms of the circulation or disturbances of the atmosphere" (Hare 1955). As such, the specification of climate itself must contain, perhaps implicitly, the clues to its explanation. The first step therefore in this study is to devise an objective method which meets these requirements. Secondly, it must be tested in the regional context.

The central problem to be faced is how to incorporate time variations into this study of spatial relationships. A solution is normally sought through some kind of statistical generalization, the simplest being the application of the arithmetic mean. Areal analysis is then comparatively straightforward and maps exist of annual averages of most meteorological variables. But these are often obviously not sufficient for describing a multidimensional variable even at the elementary level. When large seasonal variations are apparent, as with temperature, mean monthly distributions are often resorted to although only the mid-summer and mid-winter ones are used extensively. These go a long way towards describing the seasonal pattern which must ultimately be related to the earth's planetary movement, but they still mask the variability contained within them. And variability is important for, after all, the essence of a dynamic system, even though it may appear quasi-static in the long term analysis, is its continual evolution and change.

In precipitation analysis a limited kind of variability is frequently plotted. However, apart from the upper-air geostrophic wind analyses of Lahey et al. (1958), there are few examples where maps have been drawn of the standard deviation or variance of meteorological elements. With the advent of the electronic computer, time taken in calculations of this type is no longer a problem. Similarly, whilst means are a necessary basis for discussion of variability, they are seldom sufficient in themselves for representing internal correlation which is essential to explanation. Nevertheless workers persist in using means for such a purpose. For example, whilst it may be shown from average figures that heat transport by atmospheric motions between the tropical and polar latitudes must exist, the mechanism for such transport cannot be described by average winds and temperatures but only by their instantaneous covariance (see Chapt. 6). There is a need then for more knowledge of single series variability and of multiple series joint variability or covariability.

But just total variance or covariance is not sufficient. In simple problems of a single variable where observations are physically independent and normal, the mean and variance characterize the distribution completely. However, meteorological time series like most geophysical time series are not completely random: they have persistence. In other words, successive values in the series have a higher probability of being similar than they have of being quite different. Consequently, the nature of this persistence needs to be specified such as by the frequency components, or spectrum, of the variance. This aspect is often neglected but it is most important for any explanation of geophysical series. New objective techniques for the isolation of frequency components are already available to the climatologist but he is hesitant to use them. Instead he persists in adopting more subjective ways of analysis such as counting the number of cyclones and anticyclones. Almost a decade ago Hare (1957) made a plea to climatologists, as yet unanswered, for the use of "statistics, but not the arid statistics of the past generation; this is the age of Wold and Wiener, of spatial statistics and time series analysis."

The variance spectra will reveal the proportionate importance of the various scales of disturbance existing through the series. Therefore, they not only describe the variability of the series, but also refer to the specific size of the disturbing system. This goes a long way towards explanatory description especially if the statistical estimates can be related to particular physical processes. Further, since any atmospheric disturbance will involve more than one element, the cross spectra may be used in seeking associations between series. Cross spectral analysis is, in fact, equivalent to linear regression at each frequency. From the calculated spectral estimates it is possible then, as will be shown later (Chapt. 5), to construct time models of these disturbances which, with care, may be translated into spatial models and related to the atmospheric circulation.

With these preliminary comments on the problem of reducing time series to a few parameters, the aims of this investigation into the dynamic climatology of New Zealand may be stated. They are to show:

(i) how, and with what confidence, frequencies present in meteorological data may be objectively isolated (Chapt. 2),

(ii) what frequencies, within the limitations of time and data, are present in the New Zealand area (Chapts. 3 and 4), and

(iii) how these frequencies are intimately associated with one another as part of the atmospheric circulation (Chapts. 5 and 6).

In the remainder of this chapter a brief account of the historical aspects of spectral analysis will be given, followed by a discussion of the data.

1.2 Historical Aspects of Spectral Analysis

This is not the place to discuss the detailed history of the development of the mathematical theory, but briefly it may be stated that it combines two methods whose derivation may be found in recent text books on time series analysis. Where truly periodic phenomena occur the older method of direct Fourier or harmonic analysis may be applied but where there is no true periodicity, as is the usual case, Fourier analysis is conducted on the normalized lagged products of the series rendering a continuous spectrum. The latter approach is relatively new, and for a single series has been developed by Tukey (1949) and Blackman et al. (1958) from a theorem by Wiener (1930) for the application to communication engineering. Since 1949 a number of authors have investigated the mathematical aspects and the problems of applying it to other disciplines. The technique has been extended to the study of two series and the author has relied heavily upon recent papers by Goodman et al. (1961) and Jenkins (1962, 1963), who in turn based his work upon Goodman (1957).

In meteorology, interest in harmonic analysis was revived in the mid 1950's when hemispheric analyses of pressure were first possible on an operational basis. Classical Fourier analysis was a simple objective tool which could be used to separate the scales of motion (Charney 1947, Eliasen 1958). It demonstrated that in the Northern Hemisphere two scales of motion are important. First there is the quasi-stationary long wave system of about 120 degrees longitude in length and secondly the shorter rapidly moving waves of the cyclone and anticyclone (Shapiro et al. 1963). If predictable connections could have been made between the day-to-day patterns a great advance in forecasting would have been made. No simple relationships were discovered and although the technique is still most useful in scale analyses and will be referred to later, its importance in operational meteorology has again declined. A similar revival has taken place in climatology. Having used harmonic analysis in hemispheric wind studies (Lahey et al. 1958), Bryson applied the same procedure in a study of normal monthly precipitation totals in the United States (Horn et al. 1960) and in Canada (Sabbagh et al. 1962).

Estimation of a continuous spectrum in meteorology was first introduced by Panofsky (1953) and other authors to the microscale where it has had enormous success during the last decade. This work has been admirably summarized by Lumley et al. (1964). In 1955 Panofsky (1955) advocated the extension of spectral analysis into larger scale studies and followed this up with an analysis of temperature over a long time period (Griffith et al. 1956). This investigation revealed that most of the variance of temperature at University Park, Pennsylvania, was due to frequencies between 0.05 and 0.5 cycles per day (periods of 2 to 20 days) with a peak at 4 days but no particular gaps. A similar long period study of winds at Brookhaven, Massachusetts, supported by estimates at Oak Ridge, Tennessee, and

Idaho Falls, Idaho, by Van der Hoven (1957), confirmed the existence of a spectral gap between two peaks centred at 4 days and 1 minute. Landsberg et al. (1959) have analysed temperature and precipitation records for three quarters of a century from Woodstock College, Maryland. They uncovered little support for sunspot cycles, for which they appeared to be looking in the longer term data, and they found that seasonal patterns were so different that generalizations were difficult although periods of 3, 5, 7 and 15 to 25 days seemed to stand out more than others.

Time spectra have also been published of many upper air elements (Ward et al. 1961) but they reveal somewhat less than their spatial counterparts and there has been no attempt to incorporate them into dynamical theories.

The number of cross spectra appearing in the literature is extremely small. It may be felt that the cross correlations may be inferred from the single series, that they will produce nothing new, or more likely, that the computation problem is too large. Examples are cited by Panofsky et al. (1954, 1959) for the microscale but they have not extended them to longer periods. Estoque (1955) applied covariance spectral analysis to large-scale horizontal transfer problems, and found that the most important transfers of heat and momentum at 850 mb over the United States occurred in the neighbourhood of 4 days. A similar investigation, by Chiu (1960) concluded that disturbances of periods longer than

20 days were as important as those of shorter periods.

Several gaps still exist in this field. No studies of spectra other than at the microscale (Webb 1955) have been made in the Southern Hemisphere. Indeed, no hemispheric Fourier analysis is yet possible since the number of observing stations is small, in part due to the large ocean area. Moreover, the stations which do exist are concentrated in relatively small regions. Beyond Bryson's precipitation analyses, the spatial distribution of time spectra and their interrelationships have been ignored, and similarly no study of the structure of disturbances through time spectra has been attempted. The present investigation seeks to extend knowledge into these areas.

1.3 Data

The immediate problems in studying the spatial variation of any element are the selection of the data points and the estimation of their regional representativeness. In the present investigation there is no possibility of sampling since the number of recording stations is extremely small. The most frequent, uniform and reliable surface observations of pressure, temperature, dew point temperature and wind in the New Zealand region come from the synoptic stations, only 22 of which regularly report every six hours. Furthermore, all of these stations are at low levels and close to the ocean. These are therefore a very biased sample of all possible data sites. However, it is assumed that the general features of variance components may be compared station with station. This assumption is a reasonable one where advective elements dominate a particular variable but where other components, such as radiation, enter it is less defendable. In temperature analysis the radiative terms will dominate especially in the diurnal and annual harmonics. Consequently an effort has been made to make a more detailed, in terms of space, analysis of these components through the use of normals in Chapter 3. Also since the areal variation of precipitation is large, in part due to orographic effects, and annual data are readily available, a parallel analysis is performed on this element in the same chapter.

One year of coded information from the 22 synoptic stations (Table 1.3.1, Fig. 1.3.1) was purchased from the New Zealand Meteorological Service and put on punched cards at the University. Since data were not available before August 13 1962 the series ran from that date. For the five upper air stations, which observe only once per day, decoded data of height, temperature, moisture and wind at standard pressure levels were available on punched cards from the Meteorological Service and these were purchased for the same period. Data extending beyond the one year period would have been most useful especially for studying lower frequencies and for testing purposes, but the delay in obtaining such data would have deferred the commencement of the calculations and prolonged the entire study. Normals used in estimating the truly periodic components were calculated and supplied by the Meteorological Service as were additional observations of daily precipitation for six stations and solar radiation for four stations.

All programmes for the University of Canterbury's I.B.M. 1620 computer were written and run by the author. Initially this computer had only a 20K storage so that programmes were written for economy in space rather than time. Consequently SPS 1 was used instead of Fortran 1. Further, since most of the programmes were operational before extra storage was available they have been used throughout.

TABLE 1.3.1

STATION DETAILS

Station	WMO Code	S	E	Height ft.
Cape Reinga	93003	34 26	172 41	623
Kaitaia	011	35 04	173 17	264
Mokohinau	060	35 54	175 07	357
Whenuapai* (Auckland)	112	36 47	174 38	90
Tauranga	185	37 40	176 12	56
Gisborne	291	38 40	177 59	27
New Plymouth	308	39 02	174 10	158
Ohakea	401	40 12	175 23	168
Wellington	434	41 17	174 46	391
Westport	516	41 44	171 36	26
Farewell Spit	526	40 32	173 00	11
Nelson	545	41 17	173 13	24
Harewood* (Christchurch)	780	43 29	172 32	102
Puysegur Pt.	806	46 10	166 37	135
Invercargill*	844	46 25	168 19	4
Taiaroa Hds.	896	45 47	170 45	250
Campbell Is.	944	52 33	169 07	76
Chatham Is.	986	43 58	176 33W	162
Lord Howe Is.	91994	33 31	159 04	35
Norfolk Is.	91995	29 03	167 56	360
Raoul Is.	91996	29 15	177 55W	160
Nandi*	91680	17 45	177 27	16

* Radiosonde stations

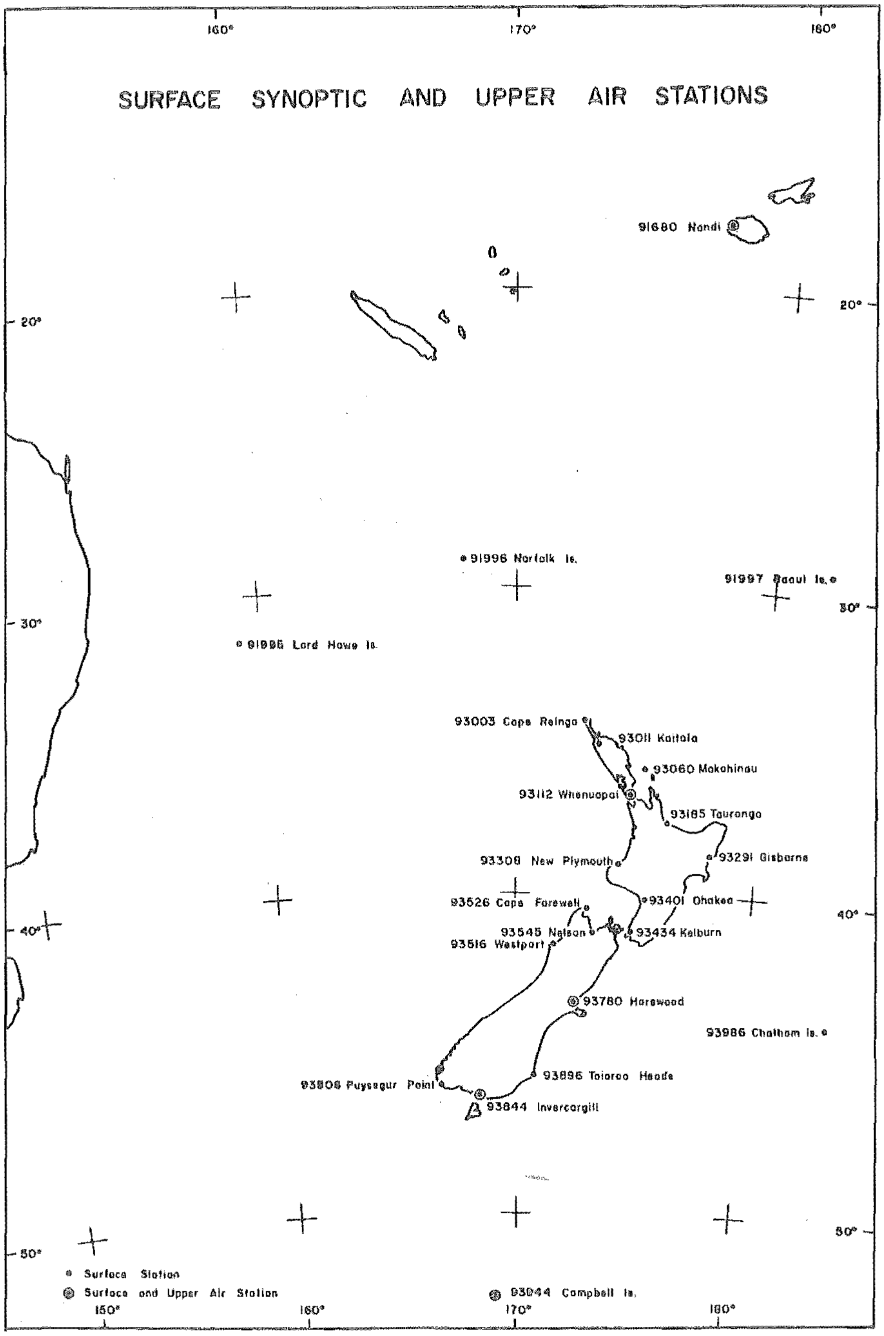
SURFACE SYNOPTIC AND UPPER AIR STATIONS
160°
170°
180°
20°
30°
40°
50°
150°
91680 Nandi
91996 Norfolk Is.
91997 Raoul Is.
Lord Howe Is.
93003 Cape Reinga
93011 Kaitaia
93060 Mokohinau
93112 Whenuapai
93185 Tauranga
93291 Gisborne
93308 New Plymouth
93401 Ohakea
93526 Cape Farewell
93545 Nelson
93434 Kelburn
93516 Westport
93780 Harewood
93986 Chatham Is.
93896 Taiaroa Heads
93808 Puysegur Point
93844 Invercargill
93944 Campbell Is.
Surface Station
Surface and Upper Air Station

Fig. 1.3.1

CHAPTER 2

METHOD OF STATISTICAL ANALYSIS

2.1 Deterministic Components

The specific equations* used for the estimation of spectra are based on the assumption that the series has a zero average and an absence of trends, so these deterministic components (Hannan 1960) must be initially calculated and removed. In other words the raw data z_t are first transformed into a new series x_t by

$$x_t = z_t - \sum \hat{\beta}_j a_{j,t} \qquad 2.1\ (1)$$

where the estimates, $\hat{\beta}_j$, are found by least squares. The only trend considered is the annual variation which is approximated by a sinusoidal curve. Also, where six-hourly observations are used, it is found convenient to remove the diurnal variation by another sinusoidal curve.

If the series, z_t , had been completely periodic a Fourier series could have been employed to estimate the spectral lines instead of the spectral densities of the later chapters. Horn et al. (1960) have shown that monthly precipitation normals are periodic and may be subjected to Fourier analysis. This procedure of approximating z_t^+ as a series of six periodic functions

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* A list of symbols is given in the Appendix.

is employed in Chapter 3, i.e.

$$z_t^+ = \sum_{j=0}^{6} A_j \cos\left(j\theta_t - \Phi_j\right) \qquad 2.1\ (2)$$

where $\theta_t = \frac{2\pi t}{12}$. The method used in the construction of the computer programme may be found in Hamming (1962). It can be seen that there is a close relationship between the coefficients for the annual waves in 2.1 (1) and 2.1 (2). In fact if 2.1 (1) were applied to all the data z_t , from which z_t^+ were calculated, the results would be the same.

After removal of the deterministic components by 2.1 (1) any necessary smoothing, as discussed later, is performed. The estimates for spectral bands are obtained following the lines of Tukey's original paper (1949) on variance spectra.

2.2 Variance Spectra

First normalized or, in this case, average lagged products are formed

$$G_h = \sum_{t=0}^{N-1} \frac{x_t\, x_{t+h}}{N-h} \qquad 0 \le h \le m \qquad 2.2\ (1)$$

where N is the number of observations and h is the lag.

Since G_h is an even, or symmetrical function of h, there is no point in calculating the negative lags but they must be incorporated in the following equations. Next, harmonic analysis is performed on the lagged products but again, since the function is even, the sine terms may be neglected. The line powers are calculated as the cosine transform of the G_h series.

$$L_{(0)} = \frac{1}{2m}\left(G_0 + G_m\right) + \frac{1}{m}\sum_{h=1}^{m-1} G_h ,$$

$$L_{(r)} = \frac{1}{m} G_0 + \frac{2}{m}\sum_{h-1}^{m-1} G_h \cos\frac{hr\pi}{m} + G_m \cos r\pi , \quad 0 \leq r \leq m$$

$$L_{(m)} = \frac{1}{2m}\left(G_0 + (-1)^m G_m\right) + \frac{1}{m}\sum_{h=1}^{m-1}(-1)^h G_h . \qquad 2.2\ (2)$$

The equations in 2.2 (2) include a band width factor so that the line powers are in terms of variance per band rather than the usual variance density. Finally these line powers are smoothed (in this case by the Hanning function). Also, if any initial smoothing is carried out inverse smoothing is performed at this stage.

$$S_{(0)} = 0.5\, L_{(0)} + 0.5\, L_{(1)} ,$$

$$S_{(r)} = 0.5\, L_{(r)} + 0.25\left(L_{(r-1)} + L_{(r+1)}\right) ,$$

$$S_{(m)} = 0.5\, L_{(m)} + 0.5\, L_{(m-1)} . \qquad 2.2\ (3)$$

In the same paper Tukey (1949) has shown that the distribution of these estimates about the population values is approximately proportional to chi-square. For example, if the population value of $S_{(r)}$ were $P_{(r)}$, then in the long run $P_{(r)} \times \frac{\chi^2_{95,\nu}}{\nu}$ would be exceeded 95% of the time where $\nu \approx \frac{2N}{m}$, the number of degrees of freedom. Similarly, $P_{(r)} \times \frac{\chi^2_{05,\nu}}{\nu}$ would be exceeded 5% of the time. Consequently there is a 90% probability that $P_{(r)}$ will lie in the region between $S_{(r)} \times \frac{\nu}{\chi^2_{05,\nu}}$ and $S_{(r)} \times \frac{\nu}{\chi^2_{95,\nu}}$.

2.3 Cross Spectra

The procedure is similar. First, lagged cross products are calculated,

$$X_h = \sum_{t=0}^{N-1} x_t y_{t+h}. \qquad -m \leq h \leq m \qquad 2.3\ (1)$$

Now the X_h series is not an even function of h so the real and imaginary parts are calculated separately.

2.3.1 Cospectrum

For the real part positive and negative lagged products are then averaged,

$$G_h = \frac{X_h + X_{-h}}{2(N-h)}. \qquad 2.3.1\ (1)$$

Equations 2.2 (2) and 2.2 (3) with inverse smoothing are then applied to give estimates of $C_{(r)}$, the cospectrum, instead of the single series spectrum. This is the spectrum of the instantaneous covariance of the two series.

2.3.2 Quadrature

The imaginary portion is produced from the differences of the average positive and negative lagged products,

$$D_h = \frac{X_h - X_{-h}}{2(N-h)}. \qquad 2.3.2\ (1)$$

These are then subjected to a sine transform to obtain quadrature line powers,

$$L_{(r)} = \frac{2}{m} \sum_{h=1}^{m-1} D_h \sin \frac{hr\pi}{m}, \qquad 0 < r < m \qquad 2.3.2\ (2)$$

and equations 2.2 (3) with inverse smoothing give estimates $Q(r)$, the quadrature spectrum.

Unfortunately, no reliable methods have yet been devised for estimating the confidence of cospectra or quadrature spectra.

2.3.3 Cross Amplitude

Analogous to the calculation of the amplitude of a single series, the so-called cross amplitude of two series at a given frequency may be found from the cospectral and quadrature spectral estimates

$$K_{(r)} = \sqrt{C_{(r)}^2 + Q_{(r)}^2} \,. \qquad 2.3.3\ (1)$$

Since $C_{(r)}$ and $Q_{(r)}$ are covariances $K_{(r)}$ is in reality the cross variance and the true cross amplitude is equal to $2\,K_{(r)}$.

2.3.4 Phase

Similarly the phase difference, $\Phi_{(r)}$, between the two series is given by

$$\Phi_{(r)} = \tan^{-1}\left(\frac{Q_{(r)}}{C_{(r)}}\right). \qquad 2.3.4\ (1)$$

The phase may be converted from an angular to a linear measure of time by

$$\tau_{(r)} = \frac{\Phi_{(r)}\, m\, \Delta t}{\pi r} \qquad 2.3.4\ (2)$$

where Δt is the time spacing of observations.

2.3.5 Coherence

Another analogy may be taken: this time with regression where the importance of the relationship is measured by the correlation coefficient. In cross spectral analysis it is called the coherence, $W_{(r)}$.

$$W_{(r)} = \sqrt{\frac{C_{(r)}^2 + Q_{(r)}^2}{S_{x(r)}\, S_{y(r)}}} \,. \qquad 2.3.5\ (1)$$

It is a measure of the correlation between x_t and y_t at a given frequency.

2.3.6 Confidence Intervals

Jenkins (1962) has shown that $K_{(r)}$, $\Phi_{(r)}$ and $W_{(r)}$ are normally distributed with the following variances

$$\mathrm{Var}\left(K_{(r)}\right) = K_{(r)}^2 \sqrt{\frac{1}{2\nu}}\sqrt{1+\frac{1}{W_{(r)}^2}} , \qquad 2.3.6\ (1)$$

$$\mathrm{Var}\left(\Phi_{(r)}\right) = \sqrt{\frac{1}{2\nu}}\sqrt{\frac{1}{W_{(r)}^2}-1} \quad \text{and} \qquad 2.3.6\ (2)$$

$$\mathrm{Var}\left(W_{(r)}\right) = \sqrt{\frac{1}{2\nu}}\sqrt{1+W_{(r)}^2} . \qquad 2.3.6\ (3)$$

As discussed after equation 2.2 (3) $\nu = \frac{2N}{m}$ except for the end values where it decreases to $\frac{N}{m}$.

2.4 Resolution, Confidence and Calculation Time

The final estimates and confidence intervals have been calculated for the frequencies

$$f_{(r)} = \frac{r}{2m\Delta t} \qquad 0 \leq r \leq m \qquad 2.4\ (1)$$

as totals for frequency bands of width $\frac{1}{2m\Delta t}$. They are plotted in this study therefore in the form of a histogram rather than a continuous density function. Also, a linear frequency scale is employed although often in meteorological applications logarithmic frequency is used with the power estimates multiplied by the frequency, thus preserving the area under the curve.

This alters the shape of the curve and the apparent relative positions of the maxima so care must be taken in interpretation. As can be seen from 2.4. (1) whilst each band is of equal width with respect to frequency it varies from $(\infty - 2m\Delta t)$ to $\left(\frac{2m\Delta t}{m-1} - 2\Delta t\right)$ in width with respect to period. Interpretation will therefore often depend upon whether interest is centred upon frequency or upon period. As another variant the logarithm of the variance spectra may be plotted, as the width of the confidence bands then remains uniform.

From the equations for confidence levels and the above discussion, it can be seen that the resolution and range of the estimates and the width of the confidence bands are all closely related. Blackman et al. (1958) recommend that the maximum lag, m , should be no greater than 10% of the data although Jenkins (1961) suggests that at least some estimates should be made to higher lags. Even for lagging to 10% the number of degrees of freedom is down to 20.

Resolution may be increased basically by increasing the number of lags, thus reducing the number of degrees of freedom unless the period of observation is likewise proportionately increased. Range can be increased at the low frequency end by the same method but the high frequency end can be changed only by reducing Δt . The latter must also be accompanied by an increase in m unless resolution is to change.

For the variance spectrum the width of the confidence band for a set probability level is completely dependent upon the number of degrees of freedom which, for a given resolution and range, is in turn entirely dependent upon the length of period of observation. This is only partly true for equations 2.3.6 (1), (2) and (3) where the value of the coherence becomes important.

A practical consideration must also be taken into account here. That is, the amount of time involved in obtaining the estimates. For co- and quadrature spectra two thirds of the time is taken up in the calculation of the lagged products and is closely related to the number of multiplications. These number

$$N(2m+1) - m(m+1). \qquad 2.4\ (2)$$

For example, with one year's data of one observation per day and a maximum lag of 40 the number of multiplications is about 28,000, a large number, even for a small electronic computer.

With these factors in mind, access to only one year's data and interest in periods between 3 days and a month the following values were selected:

$$\frac{N}{m} = 10, \qquad \Delta t = \frac{365}{N} \text{ days}, \qquad \frac{r}{2m\Delta t} = \frac{r}{73} \text{ cycles per day}$$

To give more resolution the latter was changed to $\frac{r}{80}$ so $\frac{N}{m}$ automatically becomes 9.125. This leads to 41 bands in the range 0. to 0.5 cycles per day (Table 2.4.1) with higher frequencies added if more than one observation per day is used.

TABLE 2.4.1

CENTRAL FREQUENCIES AND PERIODS
FOR WHICH SPECTRA CALCULATED

$$f_{(r)} = \frac{r}{2m\Delta t} = \frac{r}{80}$$

r	cycles per day	days per cycle	r	cycles per day	days per cycle
0	0.0000	∞	21	0.2625	3.81
1	0.0125	80	22	0.2750	3.64
2	0.0250	40	23	0.2875	3.48
3	0.0375	26.67	24	0.3000	3.33
4	0.0500	20.00	25	0.3125	3.20
5	0.0625	16.00	26	0.3250	3.08
6	0.0750	13.33	27	0.3375	2.96
7	0.0875	11.42	28	0.3500	2.86
8	0.1000	10.00	29	0.3625	2.76
9	0.1125	8.89	30	0.3750	2.67
10	0.1250	8.00	31	0.3875	2.58
11	0.1375	7.27	32	0.4000	2.50
12	0.1500	6.67	33	0.4125	2.42
13	0.1625	6.15	34	0.4250	2.35
14	0.1750	5.71	35	0.4375	2.28
15	0.1875	5.33	36	0.4500	2.22
16	0.2000	5.00	37	0.4625	2.16
17	0.2125	4.71	38	0.4750	2.11
18	0.2250	4.44	39	0.4875	2.05
19	0.2375	4.21	40	0.5000	2.00
20	0.2500	4.00			

Two other important problems must be considered before the calculations are discussed because they affect the accuracy and interpretation of the results. They are aliasing and non-stationarity.

2.5 Aliasing

Serious errors may be introduced by aliasing, that is, the folding in of variance or covariance from frequencies greater than $\frac{1}{2\Delta t}$, the Nyquist frequency. The aliased frequencies of $f_{(r)}$ are

$$2\left(\frac{1}{2\Delta t}\right)-f_{(r)},\quad 2\left(\frac{1}{2\Delta t}\right)+f_{(r)},\quad 4\left(\frac{1}{2\Delta t}\right)-f_{(r)},\text{etc.} \qquad 2.5\ (1)$$

and the estimated $S_{(r)}$, (or $C_{(r)}$ or $Q_{(r)}$) is the sum of the S (or C or Q) at all these frequencies. The inset diagram on Fig. 2.5.1 shows how an oscillation of period 1¼ days appears to be one of period 5 days when Δt = 1 day. For a chosen frequency interval, $\frac{1}{2m\Delta t}$, aliasing may be reduced only through a more detailed knowledge of x_t and y_t within the interval Δt . Observations may be taken more often and spectra calculated to higher frequencies in the hope that the power beyond the new Nyquist frequency is relatively small. If there is no interest in the higher part of the spectrum this is uneconomical. For example, if the frequency spacing is maintained and observations are taken at half the original interval the number of multiplications (from 2.4 (2)) is quadrupled.

In an attempt to estimate the magnitude of aliasing the variance spectrum from the six-hourly wind measurements

VARIANCE SPECTRUM OF S—N COMPONENT OF WIND OVER CHRISTCHURCH AT 500 mb., 5·5 km., AUGUST 1962 — AUGUST 1963

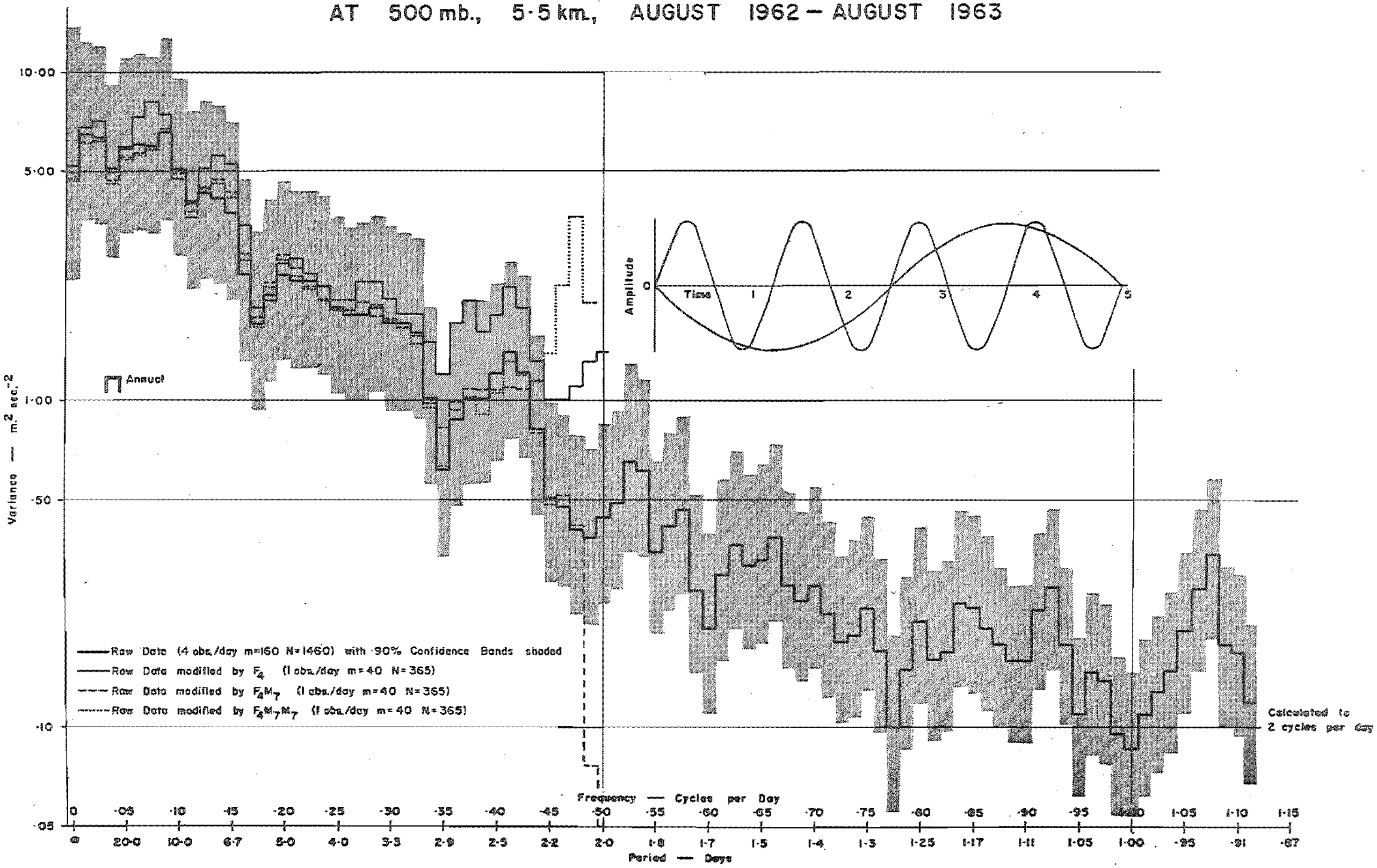

Fig. 2.5.1

at 500 mb over Christchurch is compared with the spectrum from daily observations. The diurnal component has been previously removed from the larger series although with a magnitude of 0.02 $m^2 sec^{-2}$ this is not significant. The two sets of estimates are plotted on a logarithmic scale in Fig. 2.5.1. The total variances are similar, 125 $m^2 sec^{-2}$ for the larger series and 128 $m^2 sec^{-2}$ for the smaller.

If it is assumed that the values between 0 and 0.5 cycles per day for the larger series are the best estimates of the mean spectrum, then something may be said about the reliability of the daily data spectrum. From the point of view of confidence intervals the estimates for two thirds of the spectrum, up to 0.35 cycles per day, are well within the 90% band, but in terms of magnitude the first third has the greatest deviation especially in the peaks where it amounts to over 10%. Nevertheless, perhaps more important, the shape and relative sizes of the general maxima and minima remain unchanged except for the last four frequency intervals. The reason for this is that no significant maxima or minima occur at higher frequencies and the average magnitude in that range is relatively low. A similar picture should emerge for the spectra of upper air temperature and water vapour but six-hourly observations for these variables are not available.

Instead, the surface temperature for Christchurch is analysed in the same way. Two calculations of daily data have been made. In the first, the variance of the diurnal oscillation (amounting to 16.4$^{o}C^{2}$) was removed before the midday readings were selected from the six-hourly data. In the second no diurnal fluctuation was removed. The latter series showed a smaller estimate for the zero frequency and is plotted in Fig. 2.5.2. The difference here must be due to the annual variation of the diurnal amplitude and will be discussed in section 2.7. As in Fig. 2.5.1 the total variances are similar: 13.8$^{o}C^{2}$ for the larger series and 13.9$^{o}C^{2}$ for the smaller.

Errors due to aliasing as revealed by Fig. 2.5.2 are far more serious than in Fig. 2.5.1. Again the confidence bands for sampling variations may be used as a yardstick in studying the aliasing errors which amount to 40% in magnitude. The relative importance of the peaks and valleys is maintained except at 0.25 cycles per day. The differences between the two figures may be accounted for by the bigger variances of temperature in the frequencies greater than 0.5 cycles per day. In fact, in this example, with the annual and diurnal variances removed, the variance in one year's six-hourly data is equally divided between frequencies above and below 0.5 cycles per day.

It should be remembered that the larger series spectrum may itself be aliased and that some of the temperature variance may be artificially created by the data which were rounded to the nearest degree. Such rounding would introduce a random element which would appear in the higher frequencies. The temperature

VARIANCE SPECTRUM OF SURFACE TEMPERATURE
CHRISTCHURCH, AUGUST 1962 — AUGUST 1963

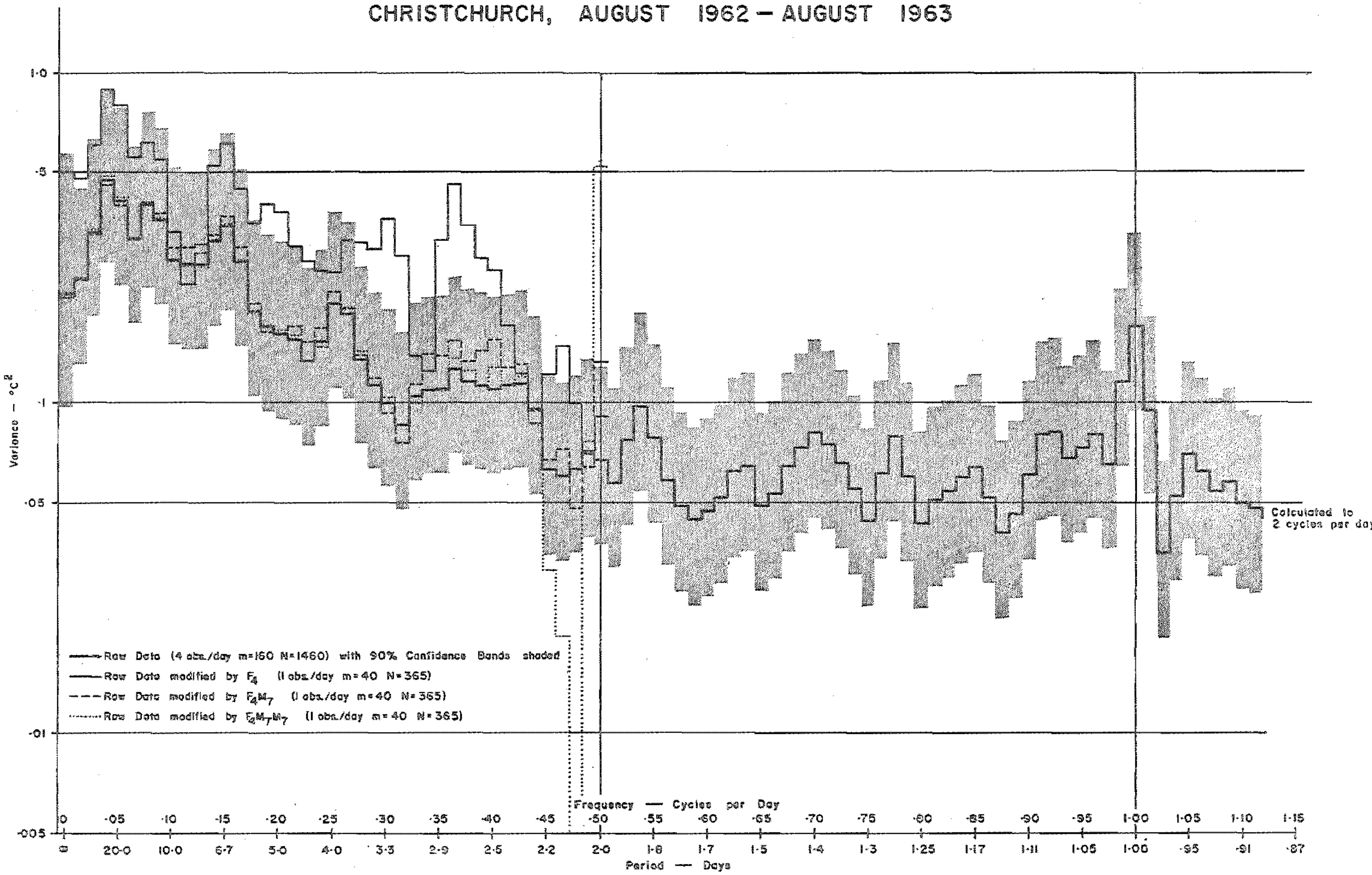

Fig. 2.5.2

spectrum does appear to be "white" in this region which would, in part, support the latter suggestion. Wind observations, on the other hand, are ten times more sensitive to time variation as is evidenced by the total variance.

It may be concluded from the above analyses, that whilst the actual magnitudes of the variance spectra for the daily data are in error by as much as 10% in the case of upper wind and 40% in the case of surface temperature, the relative magnitudes of the general maxima and minima are more reliable. Other similar comparisons support these conclusions but it is unknown whether upper air temperature will be affected in the same way.

2.6 Smoothing and Decimation

It is obvious that where more frequent data are available they should be used. Blackman et al. (1958) have suggested a technique for circumventing the computation time problem yet reducing the aliasing errors. This is accomplished by first smoothing and then decimating* the raw data. Since smoothing affects each spectral estimate differently, a proportionate adjustment known as inverse smoothing must be carried out on the final estimates.

Holloway (1958) gives a full account of smoothing

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* Blackman and Tukey (1958) use the word "decimated" to refer to the retention of only every d^{th} item - p.45.

and its results but some brief comments may be made here.

If a low pass symmetrical filter, known as an even smoothing function, is used on a set of data the effect upon the various frequencies within the data may be gauged by the filter's cosine transform. For the average, a simple example of a smoothing function, the frequency response $R_{(r)}$ is given by

$$R_{(r)} = \frac{\sin(n\pi f_{(r)}\Delta t)}{n\sin(\pi f_{(r)}\Delta t)} \qquad 2.6\ (1)$$

where n is the number of observations over which the data are averaged.

In the calculation of a spectrum a smoothed series is either multiplied by itself or with another similarly smoothed series before each frequency band is isolated. This in effect squares the frequency response. Consequently, the relationship of the unsmoothed spectrum, $S_{(r)}$, to the smoothed, $S'_{(r)}$, is given by

$$S_{(r)} = \frac{S'_{(r)}}{R^2_{(r)}} \qquad 2.6\ (2)$$

In other words, the original balance is restored through division by the square of the frequency response of the smoothing function. If the smoothing function is applied twice then the fourth power of the response function must be used in 2.6 (2).

This by itself does not aid in reducing the number of lagged products. However if, before the spectrum is estimated, the series is decimated the high frequencies will be removed and the number of calculations proportionately reduced. With every d^{th} value retained 2.6 (1) becomes

$$R(r) = \frac{\sin\left(\frac{n\pi r}{2dm}\right)}{n \sin\left(\frac{\pi r}{2dm}\right)} \qquad 2.6\ (3)$$

In the present study several different filters were tested but it was found that Blackman and Tukey's system was the most suitable. The effects of two of these systems are shown in Figs. 2.5.1 and 2.5.2. The smoothing and decimating processes are drawn symbolically in Fig. 2.6.1. M_n refers to averaging over n observations and d to the retention of every d^{th} item. The most suitable values for n and d were found to be 7 and 4 respectively. In each example M_7 was applied once and twice before decimation to produce two series.

The effect of decimation is to move the Nyquist frequency to a lower value. The retention of every fourth item produces a new spectrum which is equivalent to the first quarter of the original but with the higher ¾ folded in, or aliased. However, prior smoothing has reduced the amplitudes of these higher aliased frequencies so that they are small compared to the true magnitudes of the new spectrum. The power of the smoothing and decimation procedure may be estimated by a comparison of the frequency response of the smoothing function at the new basic frequency and at its aliased frequency. The figures in Table 2.6.1 have been adjusted so that they represent the amount of the true amplitude transferred to the new spectrum

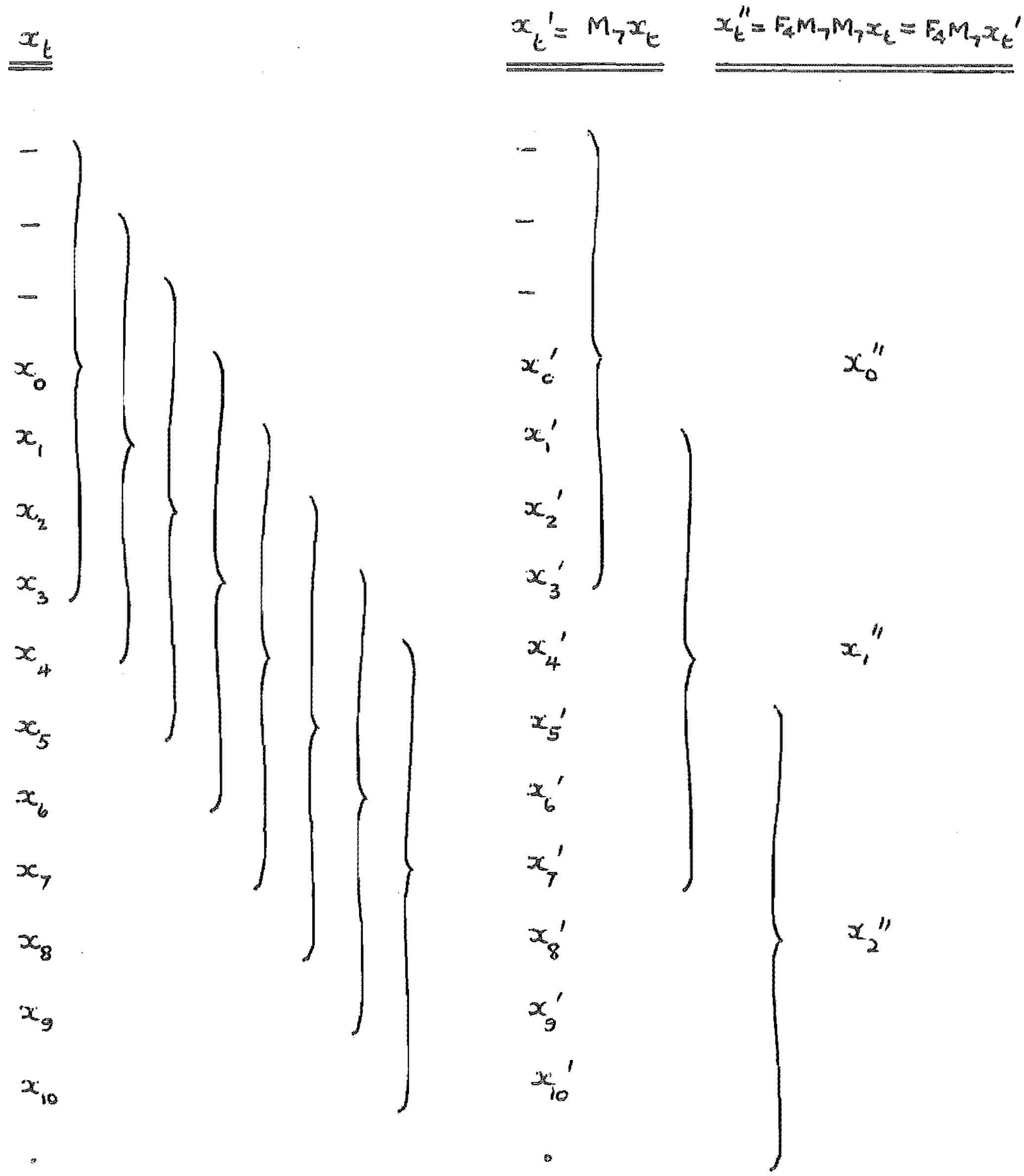


FIG. 2.6.1 SMOOTHING AND DECIMATION PROCESS

The averaging, symbolized by the brackets, takes place only over the data present, i.e. if one observation is missing the sum of the remaining six is divided by 6. The end three values are assumed to be missing. This affects only the initial two values of the x_t'' series.

TABLE 2.6.1

THEORETICAL PROTECTION FOR FREQUENCY f(r) AGAINST ALIASING FROM THE THREE SETS OF HIGHER FREQUENCIES LISTED

(r)	$2(\frac{1}{2\Delta t}) - f(r)$		$2(\frac{1}{2\Delta t}) + f(r)$		$4(\frac{1}{2\Delta t}) - f(r)$	
	$F_4 M_7$	$F_4 M_7 M_7$	$F_4 M_7$	$F_4 M_7 M_7$	$F_4 M_7$	$F_4 M_7 M_7$
0	49	2401			49	24×10^2
1	42	1780	58	33×10^2	49	24×10^2
2	36	1355	70	48×10^2	50	25×10^2
3	32	1054	85	73×10^2	50	25×10^2
4	29	837	11×10	12×10^3	51	27×10^2
5	26	677	14×10	20×10^3	53	28×10^2
6	24	556	19×10	38×10^3	55	30×10^2
7	22	464	29×10	83×10^3	58	33×10^2
8	20	391	47×10	22×10^4	61	37×10^2
9	18	335	92×10	85×10^4	64	42×10^2
10	17	290	26×10^2	68×10^5	69	48×10^2
11	16	254	28×10^3	80×10^7	76	57×10^2
12	15	224	16×10^3	24×10^7	83	70×10^2
13	14	201	20×10^2	41×10^5	94	88×10^2
14	13	182	74×10	54×10^4	11×10	12×10^3
15	13	166	37×10	14×10^4	13×10	16×10^3
16	12	154	22×10	49×10^3	15×10	24×10^3
17	12	144	15×10	21×10^2	19×10	38×10^3
18	12	136	10×10	10×10^3	26×10	68×10^3
19	11	131	75	56×10^2	38×10	15×10^4
20	11	127	57	32×10^2	64×10	41×10^4
21	11	126	44	19×10^2	14×10^2	19×10^5
22	11	127	35	12×10^2	60×10^3	36×10^6
23	11	132	28	79×10	20×10^4	39×10^9
24	12	139	23	53×10	28×10^2	78×10^5
25	12	152	19	36×10	72×10	52×10^4
26	13	173	16	25×10	30×10	93×10^3
27	14	207	13	17×10	16×10	25×10^3
28	16	265	11	12×10	92	85×10^2
29	19	371	9	86	58	33×10^2
30	24	592	8	61	38	14×10^2
31	34	1167	7	44	26	67×10
32	58	3332	6	31	18	32×10
33	148	21809	5	22	13	16×10
34	2392	5720489	4	15	9	81
35	302	910261	3	11	6	41
36	41	1648	3	7	5	21
37	12	151	2	5	3	10
38	5	24	2	3	2	5
39	2	5	1	2	2	2
40			1	1		

relative to 1, the amount of the aliased frequency allowed through. Without smoothing the decimation procedure would leave this ratio close to 1 : 1.

It can be seen that F_4M_7 and $F_4M_7M_7$ both give satisfactory results. The estimates obtained through them are very close to the basic six-hourly data spectrum at least in the first two thirds of the frequencies. The process F_4M_7 creates errors in the final two bands and $F_4M_7M_7$ in the final six. On the other hand $F_4M_7M_7$ gives in general more "accurate" estimates as might be expected from the theoretical values in Table 2.6.1. In the present calculations $F_4M_7M_7$ is used where sufficient data are available. Not only does this smoothing procedure give protection against aliasing but it considerably reduces the number of multiplications and hence time in calculating the spectrum.

It is now necessary to make a check on the aliasing errors introduced into the cross spectra. Since the calculation time is in the order of several hours for six-hourly data a daily series is compared with an $F_4M_7M_7$ modified series. As can be seen from Fig. 2.6.2 the general effect of aliasing on the cross spectra of the u and v components of wind at 500 mb over Christchurch is to increase the importance of the higher frequencies at the expense of the lower. Although coherence is small it has been reduced by 25% in the frequencies between 0.06 and 0.12 cycles per day. Despite this the general features of the "accurate"

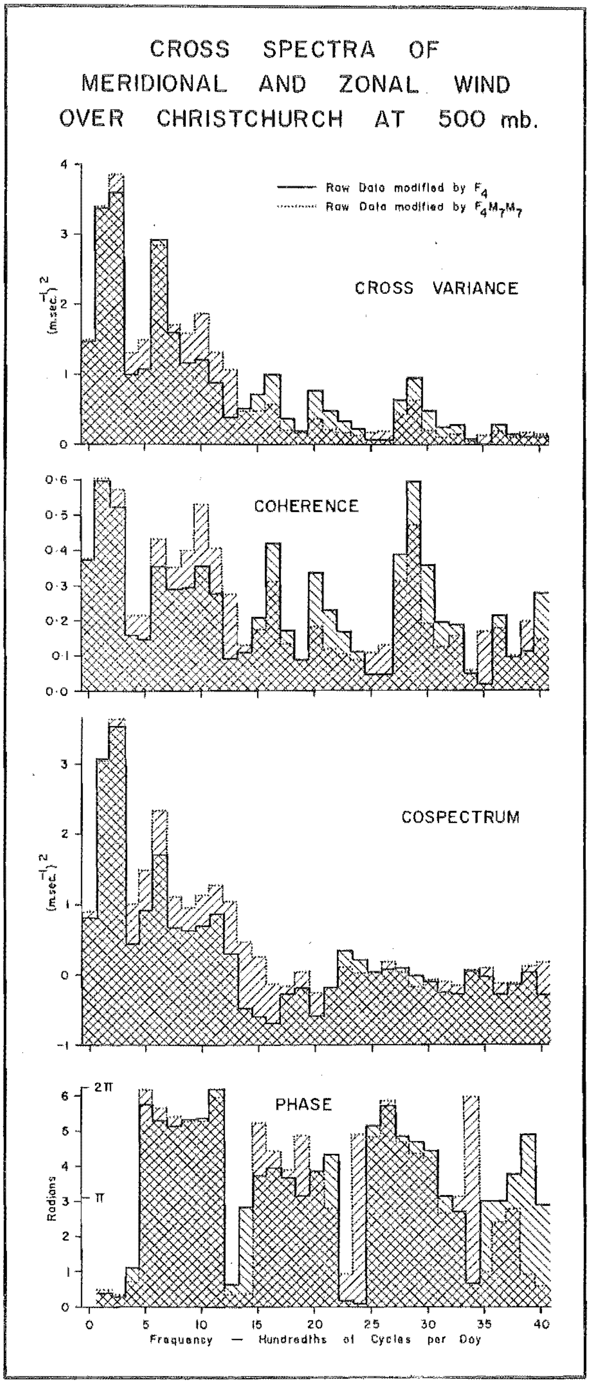

CROSS SPECTRA OF
MERIDIONAL AND ZONAL WIND
OVER CHRISTCHURCH AT 500 mb.
Raw Data modified by F_4
Raw Data modified by $F_4M_7M_7$
CROSS VARIANCE
$(m.sec.^{-1})^2$
COHERENCE
COSPECTRUM
$(m.sec.^{-1})^2$
PHASE
Radians
2π
π
Frequency — Hundredths of Cycles per Day

Fig. 2.6.2

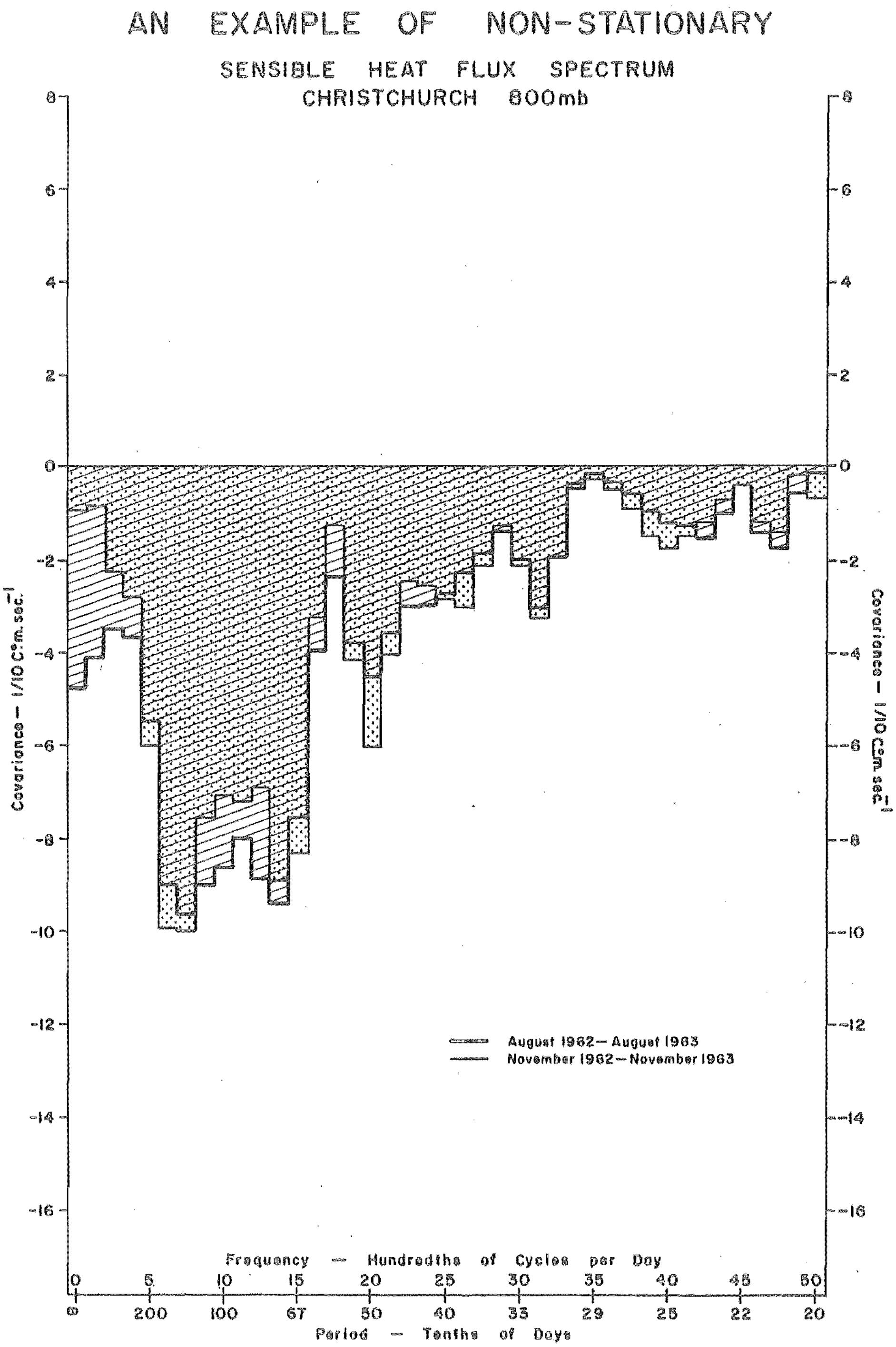
spectra appear in their aliased counterparts. Again unfortunately no check can be made on upper air temperature and wind cross spectra but it is assumed that the last conclusion applies to them also.

2.7 Non-stationarity

A further problem to be considered in the interpretation of spectra is that of non-stationarity. In a truly stationary time series the spectrum remains relatively constant (within the sampling errors) irrespective of when the observations were made. Therefore, for the confidence intervals to have any real meaning stationarity must apply. Unfortunately geophysical series are usually non-stationary so that generalizations about the fine details of one spectrum cannot be applied to a period other than the one from which that spectrum was calculated.

An example of changes which may occur in the cospectrum is drawn in Fig. 2.7.1. In each spectrum one year of data is analysed, in the first from August 1962 to August 1963 and in the second from November 1962 to November 1963.

Where the amplitude in any one frequency band changes during the period of observations the resulting spectrum is blurred. This is in addition to the smudging produced through the use of a finite set of data. A simple example is the annual change in the average diurnal amplitude of temperature which is discussed in the next chapter. The normalized lagged



AN EXAMPLE OF NON-STATIONARY
SENSIBLE HEAT FLUX SPECTRUM
CHRISTCHURCH 800mb
Covariance — 1/10 C°m. sec.⁻¹
August 1962— August 1963
November 1962— November 1963
Frequency — Hundredths of Cycles per Day
Period — Tenths of Days

Fig. 2.7.1

products of such variations are in the form of an oscillatory decay. This pattern in turn, through a cosine transform, produces a bell shaped spectrum (Barber 1961) centred on one cycle per day. The question immediately arises as to where this new annual frequency component should be placed. Obviously it should not be allowed to blur the diurnal and adjacent estimates. Nevertheless it is a different type of component from the ordinary annual oscillation although it is intimately associated with it.

The same example may be used in the discussion of a change in magnitude of a band from one set of data to another. If a year's series were divided into two parts the diurnal cycle would still be blurred, but less so than in the annual case, and the magnitudes would be different.

From these few remarks it may be concluded that where one frequency affects a second the importance of the first will be underestimated and the second will be blurred. It is possible to imagine a synoptic system, where during one period cloud damps out the diurnal amplitude and during the next clear skies allow large daily temperature variations yet the mean temperature remains the same throughout. In the analysis of the spectrum the average diurnal amplitude will be calculated but the synoptic amplitude ignored, or rather will appear as wings on the sides of the diurnal line. So, obvious processes producing non-stationarity are the annual frequency affecting all other frequencies and frequencies less than one cycle per year affecting the diurnal amplitude. There are many others.

Removal of the truly periodic components and decimation will have different effects upon the final spectrum of a non-stationary series. The diurnal amplitude, as calculated by the fitting of a sinusoidal curve to a year's data by least squares, will be the average amplitude for that period. This in general will be an underestimate in summer and an overestimate in winter. Consequently the removal of such an average will leave a weak diurnal fluctuation in summer and a weak reversed diurnal fluctuation in winter. In the calculation of the lagged products no account will be taken of the reversal and variance will remain in the diurnal frequency. A peak is visible in Fig. 2.5.2 at one cycle per day even though the strictly periodic component at that frequency has been removed.

The same residual variance will affect a decimated series. If the mean, annual and diurnal components are first removed from a temperature series, and decimation follows to select each midday value, a new annual trend will appear in the data. This effect was noted previously in the calculation of the spectrum of an unsmoothed but decimated series.

As stated above, non-stationarity limits the generality of any conclusions drawn on the line structure of a spectrum. However, the overall shape of the spectra should be more reliable. Any vast changes in maxima and minima of atmospheric spectra would be apparent in observational data. Furthermore, lack of stationarity should not affect the spatial comparisons so seriously.

Because of these influences, aliasing and non-stationarity, Blackman et al. (1958) and Jenkins (1961) have maintained that it is far more useful to show consistency between similar series than to calculate in detail a theoretical confidence band.

2.8 Missing Observations

Missing observations have been dealt with in two ways. Where the series is smoothed the missing observation is ignored and averaging is carried out on the remaining data. So long as gaps in the record are short and far apart no serious errors should be introduced. For smoothing at the beginning and end of the series the data are assumed to be missing so that the two end values will be unrepresentative but again this should not greatly influence the final spectrum. Where no smoothing is carried out missing observations are replaced by zero in the x_t series and, therefore, add nothing to the lagged products and hence to variance or cross variance spectra. The number of missing observations is generally less than 5% except at the 200 mb level where results must be regarded as more tentative. Usually data missing in meteorological series will be biased and consequently should not be ignored. Threshold values of the mixing ratio, q , are punched on the cards (New Zealand Meteorological

Service, 1963) and are used in the calculation of vapour transfer which again must introduce small unknown errors.

CHAPTER 3

DETERMINISTIC COMPONENTS

3.1 Introduction

Deterministic components in the present context will refer to the mean and truly periodic elements. As previously explained the mean and longer term trends must be removed from the data before spectral analysis is performed, or alternatively their effects must be accounted for in the resulting spectrum. In either case there is great interest in these components for the mean acts as a reference point for the variability and the true harmonics frequently account for a high percentage of the total variance. Whilst the spectrum, as analysed in the next chapter, has neither the range nor the resolution to isolate the annual and diurnal components, the fitting of sinusoidal curves to the raw data is relatively straightforward and simple.

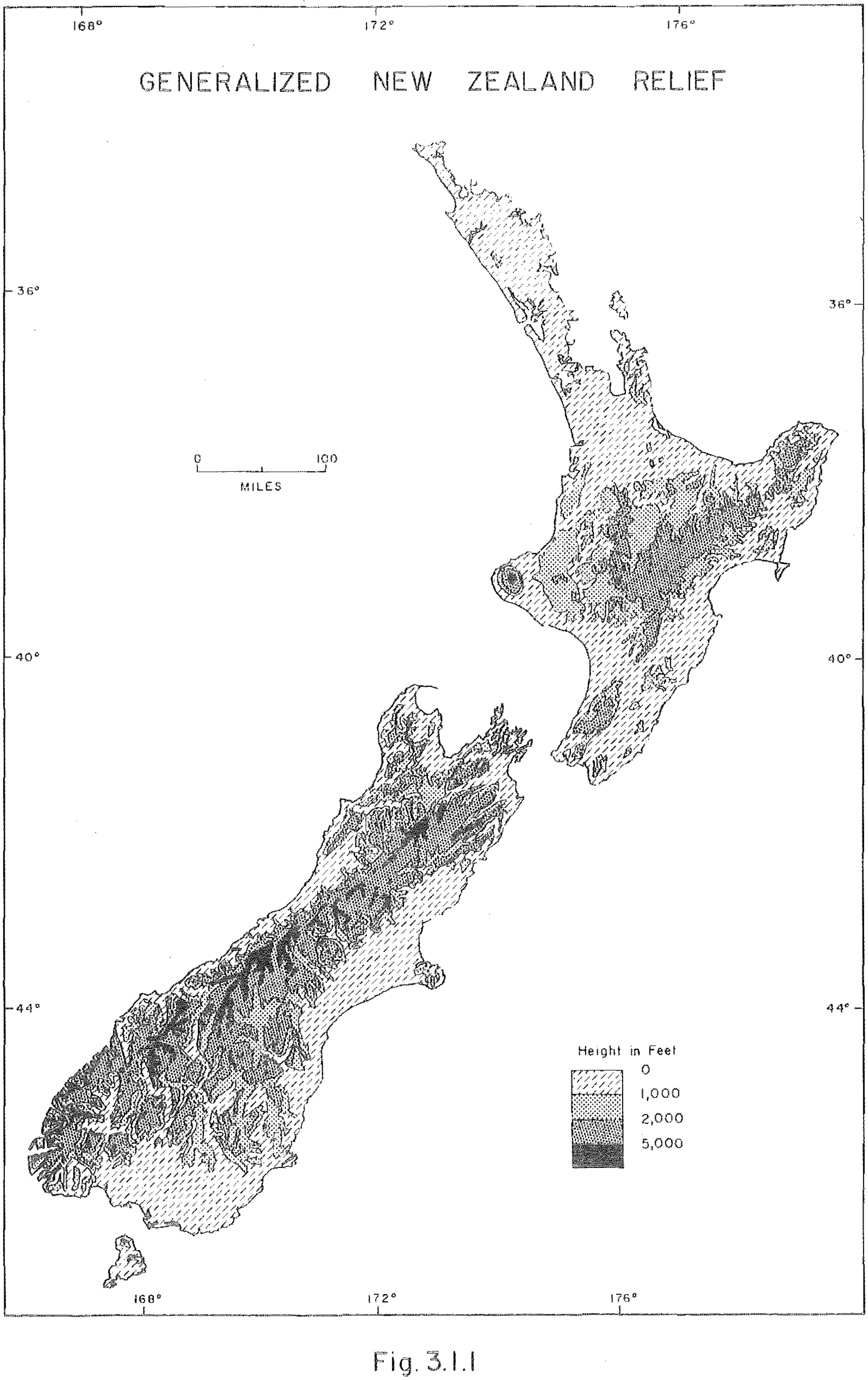
Naturally these estimates will be biased owing to the year to year variation of climate. Like the bias due to unrepresentativeness of site this needs to be evaluated. Fortunately long period mean values are readily available and the annual harmonic may be easily calculated from monthly means. Furthermore, there are far more climate stations than synoptic stations so the regional pattern may be estimated, at least in the cases of temperature and precipitation.

In this chapter the long term patterns will be reviewed and the estimated values compared with them. There is no point in mapping mean values since these already exist in a number of publications (for example Garnier 1958, McClintock 1960) but the periodic components are frequently overlooked or dealt with in a subjective manner. A generalized relief map of New Zealand has been included in Fig. 3.1.1.

3.2 Surface Temperature

Harmonic analysis of temperature here reveals only three components; the annual, the diurnal and the annual amplitude of the diurnal variation. Although there are 160 stations recording daily maxima and minima, virtually all are located on low land. They are plotted with the precipitation stations on Fig. 3.2.1. The stations are not homogeneous from the point of view of data period but most have recorded for more than 30 years.

Average daily maxima and minima for each month were used in the calculations. For the annual harmonic these maxima and minima were averaged. It is well known that such a procedure is not a very accurate method for estimating the true mean for a specific day or month, but when used in relation to other means calculated in the same way the resulting error will be small. The minimum amplitudes (Fig. 3.2.2) are to be found on the western coasts which are exposed to prevailing winds from the Tasman Sea. Even so it



168°
172°
176°
GENERALIZED NEW ZEALAND RELIEF
36°
40°
44°
0
100
MILES
Height in Feet
0
1,000
2,000
5,000

Fig. 3.1.1

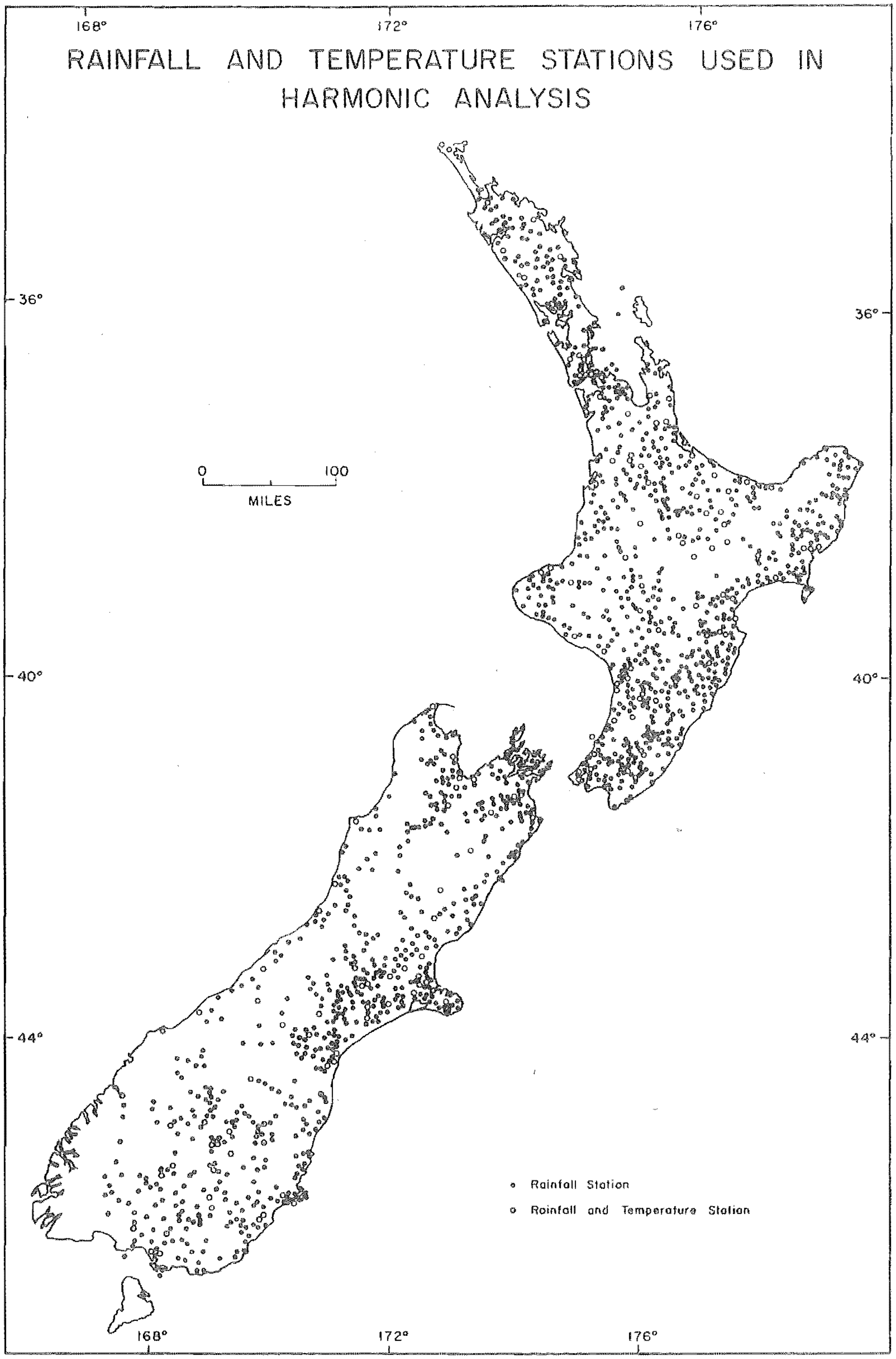

168°
172°
176°
RAINFALL AND TEMPERATURE STATIONS USED IN HARMONIC ANALYSIS
36°
40°
44°
0
100
MILES
Rainfall Station
Rainfall and Temperature Station

Fig. 3.2.1

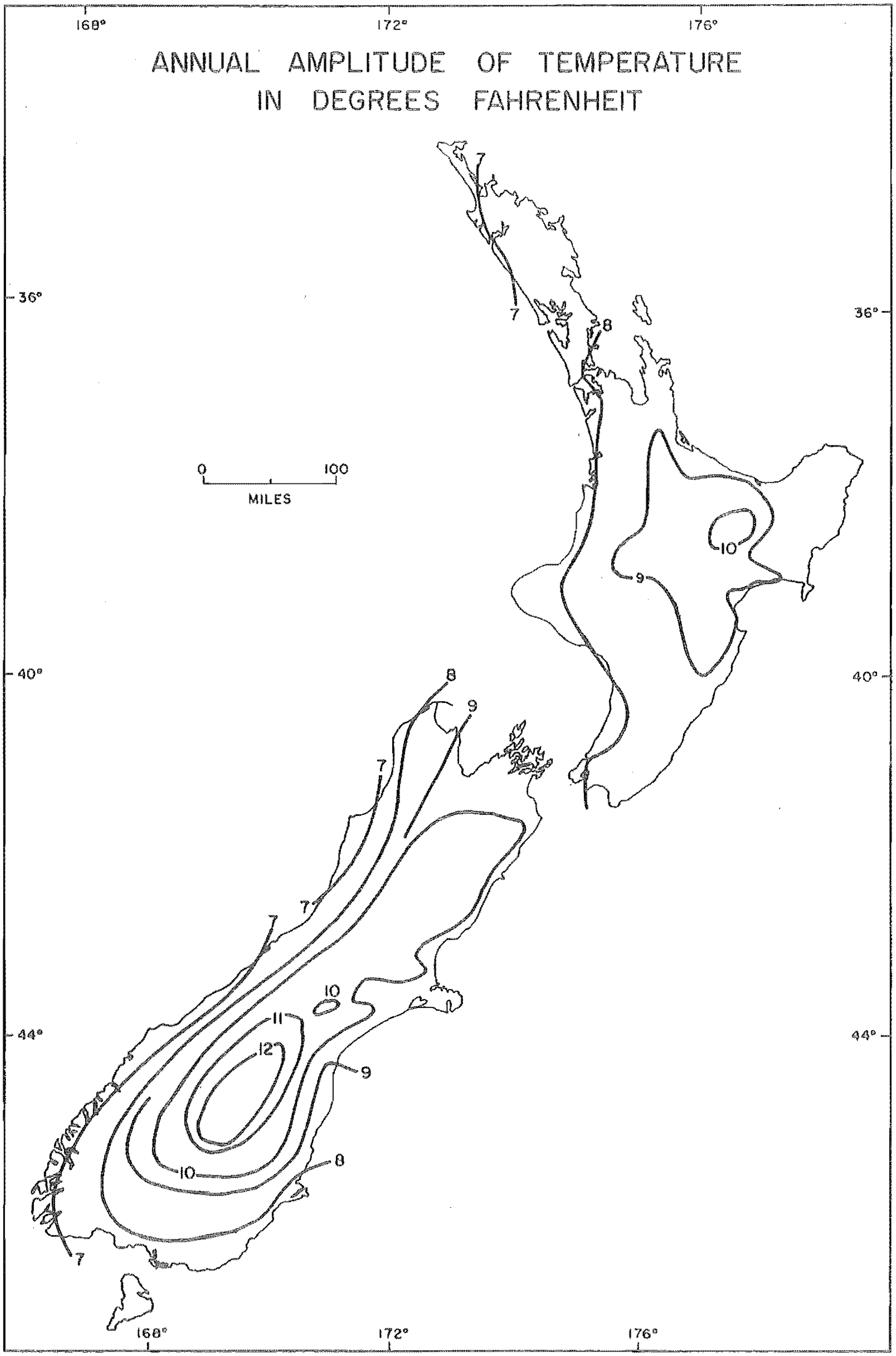

ANNUAL AMPLITUDE OF TEMPERATURE
IN DEGREES FAHRENHEIT
168°
172°
176°
36°
40°
44°
0
100
MILES
7
8
9
10
11
12

Fig. 3.2.2

amounts to a range of 12°F or 7°C. There appears to be no general trend with latitude with both North Cape and Southwestland experiencing the same range although it increases towards the central coasts. Eastern coasts record a similar though less marked influence of the ocean. Maximum amplitudes are to be found in protected regions inland. Central Otago, which experiences clear skies over most of the year, has the highest range of 25°F or 14°C. Since the maxima for most stations occur in January it is not possible to distinguish any regional pattern from the phase angles of monthly data.

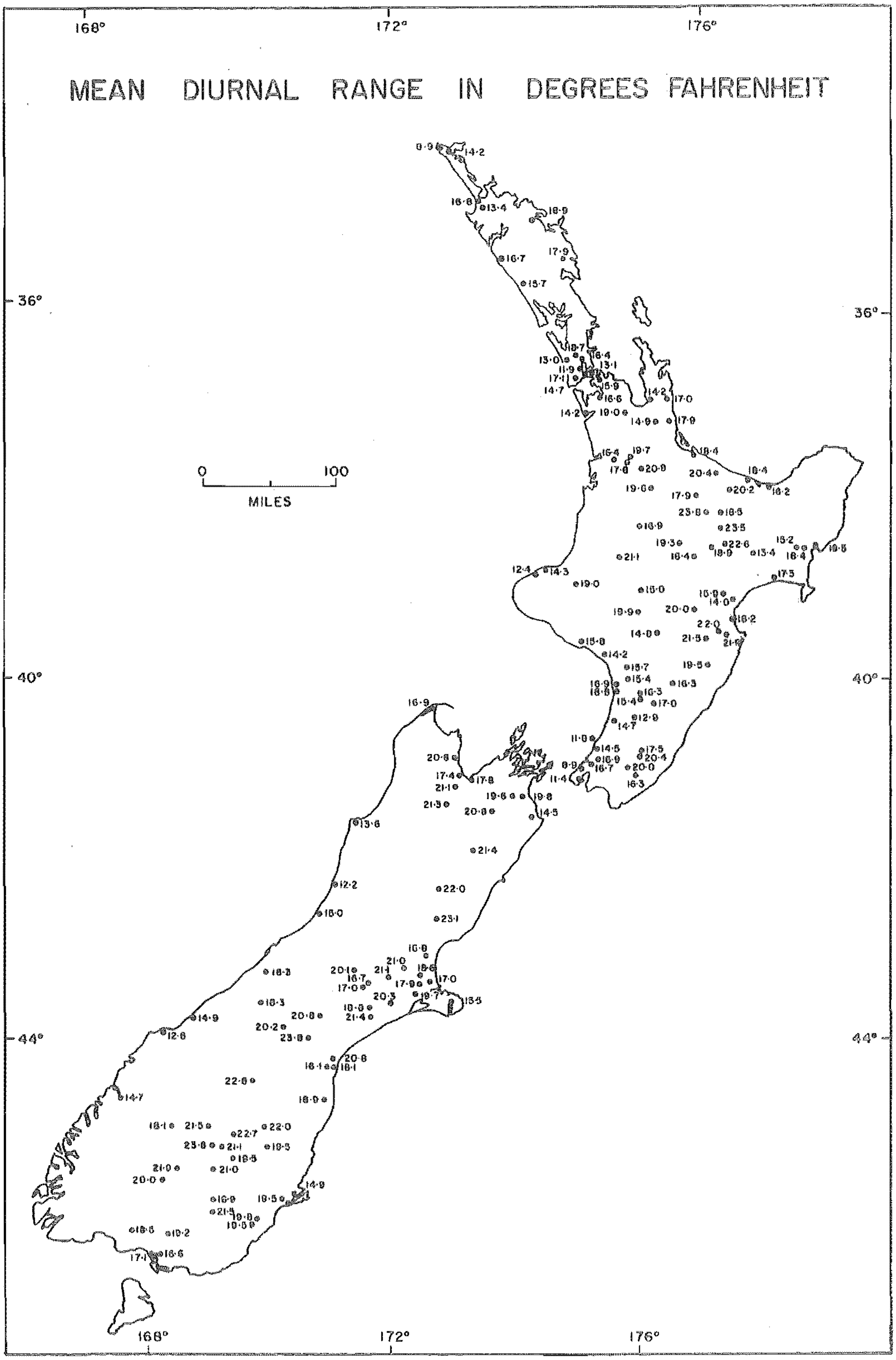
This is not so in the case of the six-hourly data. In the year 1962-63 there is a definite variation in the date of the maximum temperature. The South Island stations reach a maximum in early January, the North Island in late January and the oceanic stations to the north in February (Table 3.2.1). The amplitude pattern is similar with an increase from west to east, and northern and southern stations recording the minimum variation.

The mean diurnal range, as calculated from the differences of the mean maxima and minima, shows a less distinct pattern (Fig. 3.2.3). Coastal areas still record the lowest range and inland areas the highest, but station site now plays an important role. Maximum and minimum temperatures will be affected more by local relief, vegetation and exposure. For example, there is a 1.6°F difference between the ranges at Christchurch

TABLE 3.2.1.

DETERMINISTIC COMPONENTS OF SURFACE TEMPERATURE (°C)

Station	No. of Obs.	Mean	Normal	Diurn. Amp.	Ann. Amp.	Date of Max.	Ann. Amp. from Normals
Cape Reinga	1439	15.2	15.3	1.3	3.0	Feb. 6	3.4
Kaitaia	1445	15.2	15.2	2.7	3.6	Jan.29	3.9
Mokohinau	1430	15.8		1.3	3.7	Feb. 1	
Whenuapai	1455	14.7	14.0	3.6	4.2	Jan.23	4.1
Tauranga	1089	14.8	13.9	4.1	4.5	Jan.21	4.5
Gisborne	1439	13.9	13.8	3.5	4.6	Jan.21	4.8
New Plymouth	1456	13.3	13.1	2.7	4.1	Jan.18	3.9
Ohakea	1455	12.9		3.1	4.6	Jan.17	4.5
Wellington	1457	12.4	11.8	1.7	4.0	Jan.19	4.1
Westport	1433	12.6	11.7	3.0	4.2	Jan.15	3.7
Cape Farewell	1434	13.5		3.0	4.3	Jan.17	4.9
Nelson	1451	12.3	12.5	3.6	5.6	Jan.13	5.2
Christchurch	1450	11.3	11.4	3.8	5.7	Jan.10	5.7
Puysegur Pt.	1439	11.0		1.0	3.4	Jan.13	
Invercargill	1453	9.6	9.6	3.0	4.8	Jan.10	4.3
Dunedin	1450	10.2	10.8	1.4	4.0	Jan.14	3.9
Campbell Is.	1443	7.2	7.0	0.7	2.4	Jan.18	
Chatham Is.	1440	11.5		1.2	3.4	Jan.25	
Lord Howe Is.	1346	18.8	19.2	1.3	3.6	Feb.27	
Norfolk Is.	1400	18.6	18.8	1.3	3.0	Feb.21	
Raoul Is.	1447	18.8	18.8	1.5	2.6	Feb.12	
Nandi	1434	24.8	24.8	3.9	1.2	Feb.12	



MEAN DIURNAL RANGE IN DEGREES FAHRENHEIT
168°
172°
176°
36°
40°
44°
0
100
MILES

Fig. 3.2.3

airport and city. The airport is open, reflecting the general conditions of the whole Canterbury Plains. During the day maximum absorption of solar radiation takes place at the ground surface and at night maximum long wave radiation is possible. The city station stands in the Botanical Gardens and is surrounded by 100 ft. trees. Solar radiation is absorbed over a greater depth of the atmosphere and at night the trees reduce outward radiation by absorption and reradiation. A similar comparison of stations close together may be made in a mountain situation. At Cass (43 02S 171 46E) temperatures are recorded on a floor of a relatively flat basin as well as in a steeply sloping valley 800 ft. higher and less than one mile away. Midday temperatures are similar but the lower station may record 20°F lower temperature in winter at night when air drainage is dominant. The increase in diurnal range inland, then, is in part due to the fact that all stations in highland areas are situated in relatively low positions, and are thus subject to the temperature lowering effect of air drainage at night. Again the six-hourly data show no deviation from this pattern although values are smaller since neither the daily maxima nor minima are included.

Another interesting harmonic is that associated with variation of the mean diurnal range. This has been discussed previously since it tends to blur the spectrum. The mean range will be estimated accurately within the statistical limits of the procedure but neighbouring frequencies will be greater than their true values. For a time series the two annual components and the diurnal component may be approximated by sinusoidal curves as before and equation 2.1 (1) becomes,

$$x_t = z_t - \left[\hat{\beta}_0 + \hat{\beta}_1\left\{1 + \hat{\beta}_3 \cos\left(\theta_{2,t} - \Phi_3\right)\right\}\cos\left(\theta_{1,t} - \Phi_1\right) + \hat{\beta}_2 \cos\left(\theta_{2,t} - \Phi_2\right)\right] \quad 3.2\ (1)$$

where the subscripts 1, 2 and 3 refer to the diurnal, mean annual and daily range annual amplitudes respectively. The estimation of the $\hat{\beta}$'s is not as straightforward as before and must be accomplished by an iterative method.

For the normals a simpler procedure is adopted. The differences between the mean maxima and mean minima are analysed as in equation 2.1 (2). This, at the same time, gives the mean diurnal range of Fig. 3.2.3. Again there is a strong similarity between the amplitude of the diurnal range (Fig. 3.2.4) and the annual amplitude. In general there is an area of minimum along the coast with an increase inland. The amplitudes are smaller here and since the temperatures are given only to the nearest degree the phase angles are in some cases unreliable. Nevertheless, it is clear that maximum values of the diurnal variation occur in December or January. In other words solar radiation is still the controlling factor. The mean values agree qualitatively with those published (W.M.O. 1962).

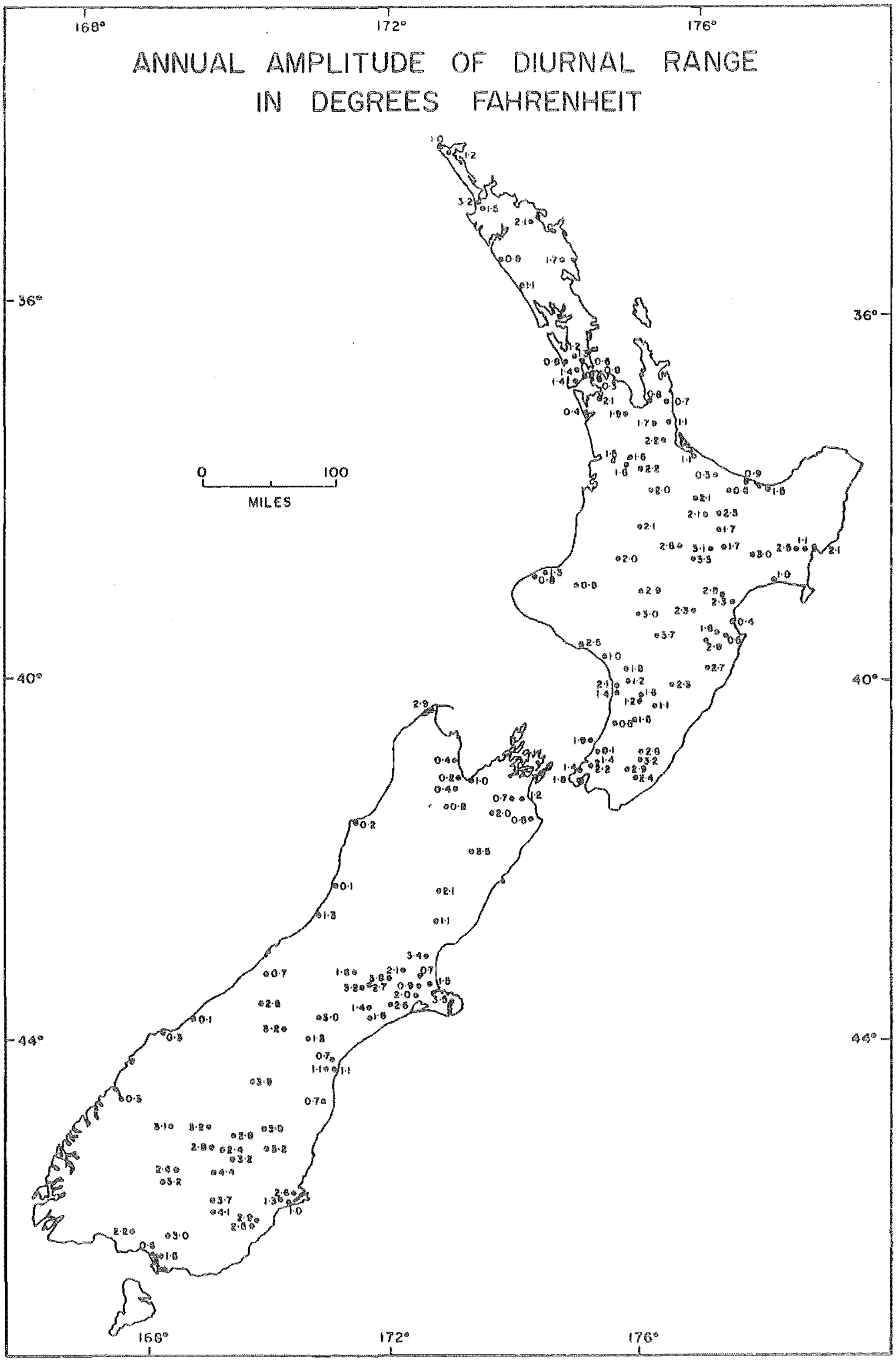

ANNUAL AMPLITUDE OF DIURNAL RANGE
IN DEGREES FAHRENHEIT
168°
172°
176°
36°
40°
44°
0
100
MILES

Fig. 3.2.4

3.3 Solar Radiation

Solar radiation is measured at only four stations in the area studied and no normals are yet available. Daily totals have been analysed and are listed in Table 3.3.1. The average daily totals over the year's record show the expected decrease with increasing latitude, the effect of increasing average zenith angle of the sun's beam. But the decrease is not smooth and Christchurch records a far greater average than the normal latitudinal decrease would imply. It is evidence of the fact that the atmosphere is more transparent to solar radiation at Christchurch than at the other stations. The annual amplitudes similarly reveal variations which must be related to regional differences of, for example, cloud amount and frequency.

TABLE 3.3.1

DETERMINISTIC COMPONENTS OF RADIATION (gcal/day)

Station	No. of Obs.	Mean	Ann.Amp.	Date of Max.
Nandi	365	462.5	91.5	Dec. 3
Whenuapai	363	351.0	189.8	Dec.23
Christchurch	365	347.4	245.9	Dec.21
Invercargill	361	309.6	234.9	Dec.21

3.4 Precipitation

Initially calculations were made on a sample of 100 rainfall stations but results, as in the case of the temperatures, were so variable from station to station that only general isopleths could be drawn. No complex method of sampling could change this situation for it is equivalent to an attempt, in the time dimension, to measure high frequency components with a sensitive instrument but with long periods between observations. If the high frequencies are required, there is no alternative to the taking of more frequent observations. From the study of a small region it was found that sufficient information for mapping at a scale clearly visible in the data required the inclusion of 80% of all possible stations. Consequently, the normals of over 1700 stations from the whole of New Zealand were analysed. Many of the normals are suspect and recognized only as preliminary estimates by the Meteorological Service. However, it was felt that with a distribution of stations as shown in Fig. 3.2.1 the true regional pattern would be brought out despite some inaccurate data. Even so, the concentration of stations in the Southern Alps and south is low and the location of isopleths is less certain there.

Most of New Zealand receives over 40 inches total annual precipitation with elevated areas recording more. Over 200 inches fall on a large area of the Southern Alps whilst the driest areas of the country are to be found in the east to the lee of the mountains. For example, a wide area of Central Otago receives less than 20 inches of rainfall per year.

The total variability of this annual rainfall, as recorded by the monthly normals, has been mapped in Fig. 3.4.1. It should be realized that the true variability has been considerably reduced through taking averages. Moreover, the frequency components are reduced unevenly as shown by equation 2.6 (1). A number of studies have been made of the year to year variations in precipitation such as Seelye (1950) and de Lisle (1956) and will not be discussed here. Also, most reviews of regional climates (Garnier 1958) contain references to the seasonal character of precipitation. However, usually there is little attempt to estimate the magnitude of the annual or semi-annual variations, and the date of the maximum falls is referred to a broad period rather than a particular month. The present analysis seeks to make values of these quantities more objective and precise.

In general there is a close relationship between total variance and total rainfall. This might be expected in the simple situation where the same processes affect high and low rainfall total areas. A variation in the strength of these hypothetical processes from month to month would affect the areas proportionately. Deviations from this ideal might be contemplated when the processes and systems producing them differ from region to region.

By equation 2.1 (2) the amplitudes and the dates of the maxima of six harmonics may be isolated. Only the first of these will be discussed in any detail since it is very difficult to find simple physical mechanisms

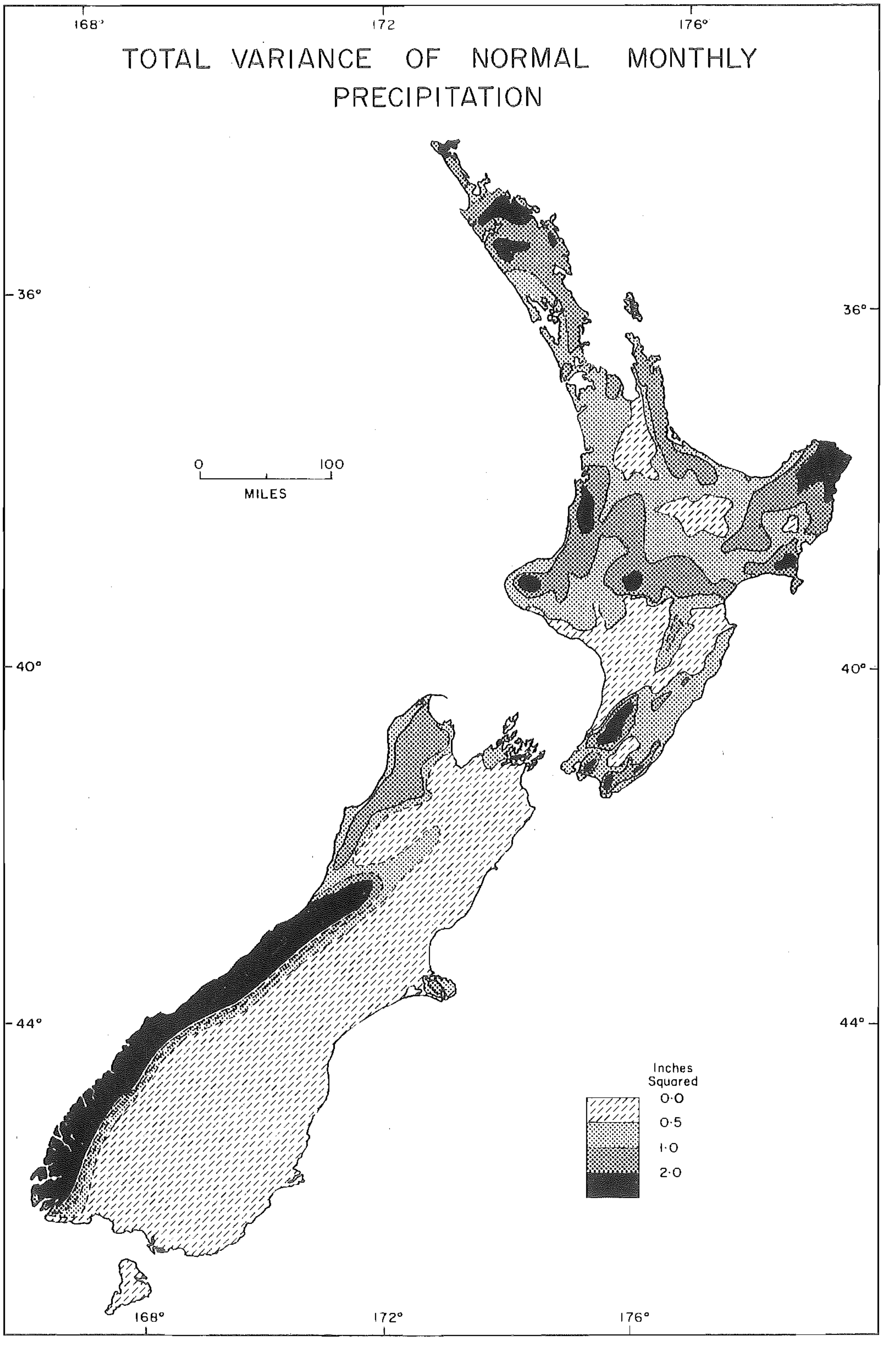

168°
172
176°
TOTAL VARIANCE OF NORMAL MONTHLY PRECIPITATION
36°
36°
0
100
MILES
40°
40°
44°
44°
Inches Squared
0·0
0·5
1·0
2·0
168°
172°
176°

Fig. 3.4.1

which can account for the others. Where two maxima occur about six months apart the second harmonic brings this out clearly. For example, Jacksons Bay (Table 3.4.1) experiences, on the average, spring and autumn maxima which account for half of the total variation. De Lisle (1956) explains the spring maximum through reference to strong pressure gradients, hence winds, which exist in that season. A similar argument may be used for the autumn maximum. In contrast at Mt. Holdsworth, whilst the annual harmonic dominates, the sixth harmonic ranks second accounting for 28% of the variance. The amplitude of 1.2 inches for this bi-monthly period is significant but a physical explanation of its occurrence is not easily found.

The percentage of total variance in the annual harmonic is mapped in Fig. 3.4.2. This component dominates over the eastern and western thirds of the North Island with an area of lesser importance down the centre. In the South Island the pattern is more complicated. The Alps, except for the south, have a remarkably low annual amplitude although this may, in part, be due to inaccurate measurement of snowfall. The amplitude of this harmonic which is directly related to the total variance and percentage of variance is given in Fig. 3.4.3. This should be referred to when the significance of the phase angle is required. The phase angle in terms of date shows that in general there is a progression through areas of high amplitude from a June maximum in the northern North Island to a

TABLE 3.4.1

PRECIPITATION ANALYSIS FOR TWO STATIONS

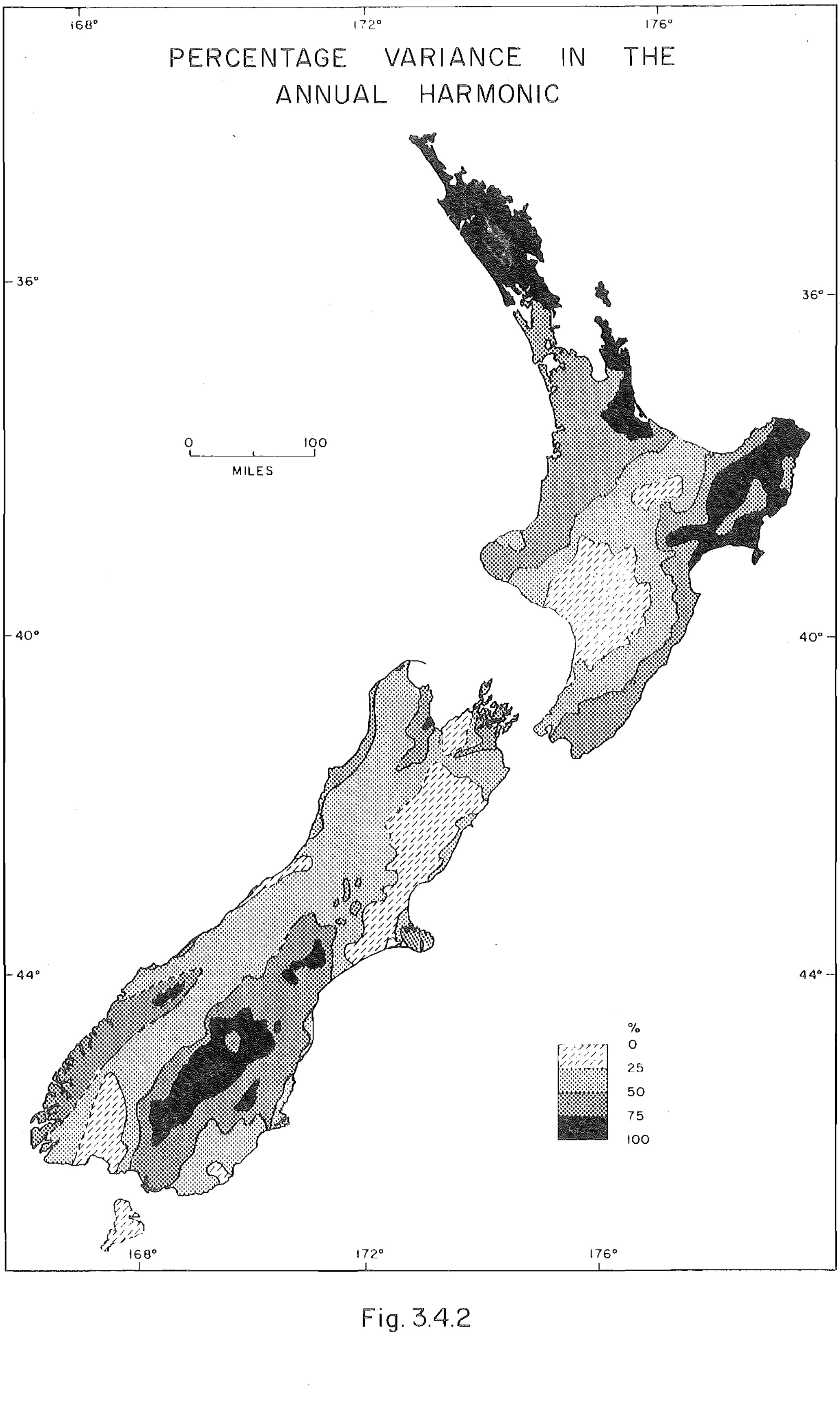
Jacksons Bay 43 59S 168 37E 25 ft. Mt. Holdsworth 40 54S 175 26E 2300 ft.

Precipitation	J.	F.	M.	A.	M.	J.	J.	A.	S.	O.	N.	D.	Year
Jacksons Bay	15.0	13.6	15.5	17.7	14.6	11.2	10.1	15.0	15.5	17.3	15.5	15.5	176.5
Mt. Holdsworth	11.3	11.3	9.4	12.0	16.0	15.7	15.1	15.3	13.1	15.1	11.1	12.8	158.2

Harmonics

	1			2			3		
Analysis	% of Var.	Amp. inches	Phase degrees	% of Var.	Amp. inches	Phase degrees	% of Var.	Amp. inches	Phase degrees
Jacksons Bay	22	1.5	343	49	2.2	166	11	1.0	309
Mt. Holdsworth	49	22	189	8	0.9	292	9	1.0	34

	4			5			6			Total
	% of Var.	Amp. inches	Phase degrees	% of Var.	Amp. inches	Phase degrees	% of Var.	Amp. inches	Phase degrees	Var. ins. sq
Jacksons Bay	1	0.3	56	1	0.3	18	15	0.5	180	5.0
Mt. Holdsworth	2	0.5	83	4	0.6	226	28	1.2	180	5.2



168°
172°
176°
PERCENTAGE VARIANCE IN THE ANNUAL HARMONIC
36°
36°
0
100
MILES
40°
40°
44°
44°
%
0
25
50
75
100
168°
172°
176°

Fig. 3.4.2

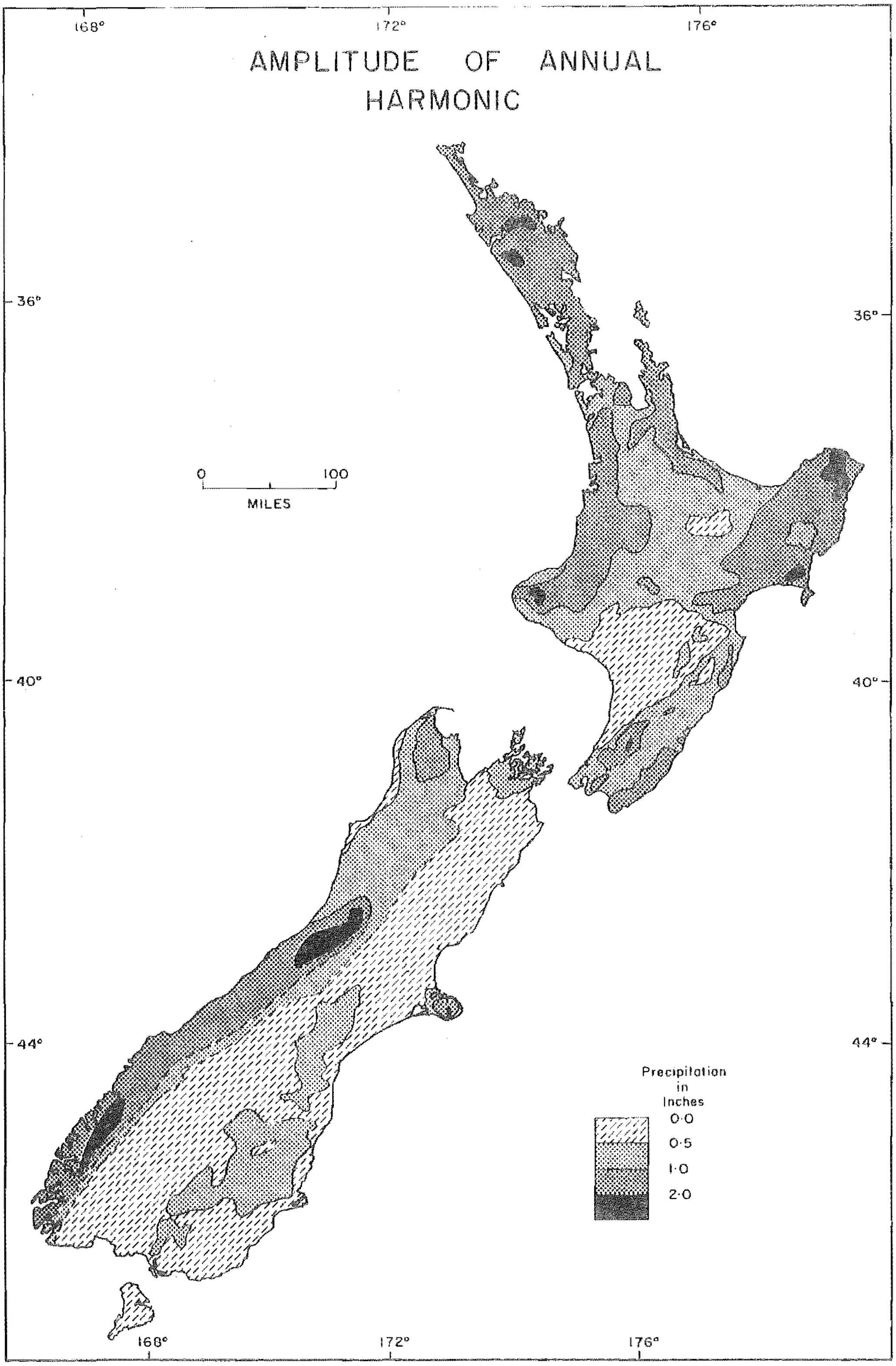

AMPLITUDE OF ANNUAL HARMONIC
168°
172°
176°
36°
40°
44°
0
100
MILES
Precipitation in Inches
0·0
0·5
1·0
2·0

Fig. 3.4.3

January maximum in the south of the South Island. Variations from this pattern are to be observed in the east (Fig. 3.4.4) where the June maximum extends southwards to Christchurch. The winter maximum throughout the eastern region suggests possible similarities in the precipitation producing mechanisms, although amplitudes are relatively small in the south.

Analysis of the daily precipitation is more difficult to interpret. Unlike temperature the daily precipitation is not distributed normally about its average value. It is characteristically positively skewed with a sharp cut-off at zero. Consequently the average has little meaning until it is converted into a monthly or annual total. Similarly the annual amplitude is insignificant when measured in units of day^{-1}. Table 3.4.2 lists calculations from daily data for six stations. Since monthly totals of precipitation are far more variable from year to year than are monthly average temperatures the results of this section are unreliable.

TABLE 3.4.2

DETERMINISTIC COMPONENTS OF PRECIPITATION (inches/day)

Station	No. of Obs.	Mean x 365	Ann.Amp.	Date of Max.
Nandi	365	74.57	0.15	Feb. 3
Whenuapai	365	55.04	0.04	Aug.20
Christchurch	365	24.00	0.02	June 18
Arthurs Pass *	365	170.44	0.05	Aug.21
Invercargill	365	40.09	0.01	Feb.21
Campbell Is.	365	45.41	0.01	June 17

* 42 57S 171 34W 2425 ft.

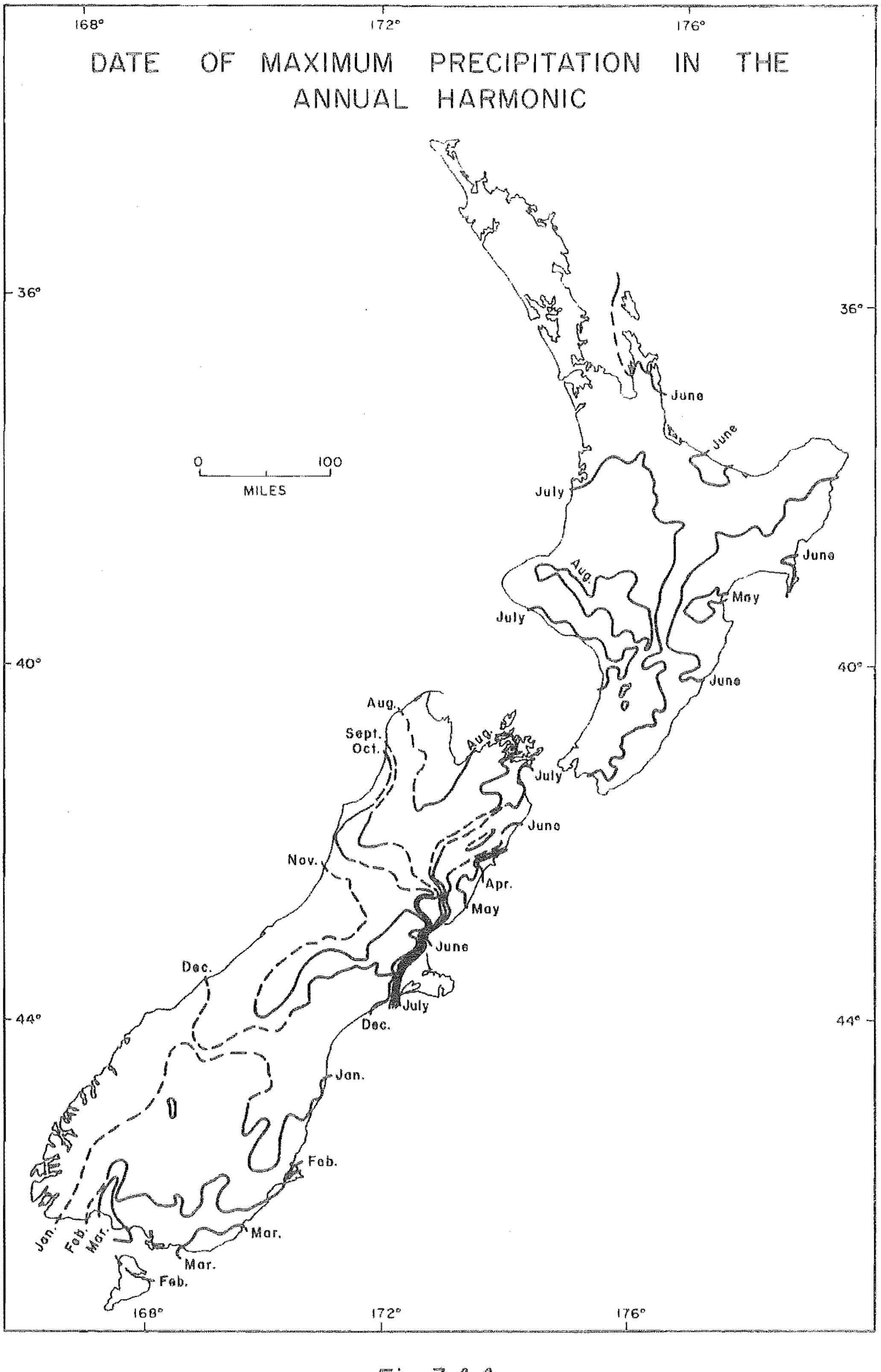

168°
172°
176°
DATE OF MAXIMUM PRECIPITATION IN THE ANNUAL HARMONIC
36°
40°
44°
0
100
MILES
June
June
July
June
Aug.
May
July
June
Aug.
Sept.
Oct.
Aug.
July
June
Nov.
Apr.
May
June
Dec.
July
Dec.
Jan.
Feb.
Mar.
Jan.
Feb.
Mar.
Mar.
Feb.

Fig. 3.4.4

3.5 Surface Dew Point Temperature

Dew point temperature has been used to indicate variations in the amount of water vapour in the surface air. Since the saturation mixing ratio is dependent upon pressure as well as temperature, variations in pressure will be incorporated into the spectra and deterministic components. However, since the dependence on pressure is considerably smaller than dependence upon temperature it is here neglected. For example, for a constant mixing ratio a change in pressure from 1000 mb to 950 mb will increase the dew point temperature 0.7°C at 0°C and 0.8°C at 10° and 20°C. More serious is the non-linearity of the dependence upon temperature. This has the effect of emphasizing changes in low moisture content relative to changes in high moisture content. In other words winter average values and variations will receive extra emphasis.

The means (Table 3.5.1) reveal a general latitudinal decrease with western exposed sites, such as Puysegur Point, recording slightly higher values than the average for the latitude. The diurnal range, which reaches a maximum close to the time of maximum dry bulb temperature, is small. On the other hand the annual amplitude, although still small, is of the same order of magnitude as the ordinary temperature. In other words the diurnal range of moisture content is small compared to the annual, whilst the effect on the relative humidity is the reverse with the maximum range occurring during a day especially in the winter months.

TABLE 3.5.1

DETERMINISTIC COMPONENTS OF SURFACE
DEW POINT TEMPERATURE (°C)

Station	No. of Obs.	Mean	Diurn. Amp.	Ann.Amp.	Date of Max
Cape Reinga	1440	12.9	0.5	3.2	Feb.12
Kaitaia	1445	11.9	0.4	3.0	Jan.30
Mokohinau	1428	12.0	0.4	3.4	Jan.27
Whenuapai	1454	10.9	0.7	3.3	Jan.24
Tauranga	1087	10.3	0.8	3.9	Jan.16
Gisborne	1436	9.8	0.9	3.7	Jan.20
New Plymouth	1456	10.0	1.0	3.7	Jan.18
Ohakea	1454	8.9	0.8	3.7	Jan.18
Wellington	1457	9.0	0.3	3.6	Jan.18
Westport	1431	8.8	1.0	4.2	Jan.15
Cape Farewell	1435	10.3	1.0	3.6	Jan.16
Nelson	1451	8.3	1.0	4.6	Jan.12
Christchurch	1448	7.2	1.0	4.4	Jan.14
Puysegur Pt.	1438	8.1	0.5	2.8	Jan.14
Invercargill	1455	6.1	1.3	3.7	Jan.12
Dunedin	1450	6.9	0.6	3.9	Jan.13
Campbell Is.	1442	4.8	0.3	2.6	Jan.12
Chatham Is.	1440	8.9	0.1	2.6	Jan.21
Raoul Is.	1447	15.7	0.3	3.0	Feb.14

3.6 Surface Pressure and Winds

Again there is a strong latitudinal variation. Maximum values of mean pressure of 1016 mb are recorded at about 30 S at the centre of the Subtropical High. Pressures fall away to the north into the equatorial low and to the south into the circumpolar low at about 65 S (Palmer 1962). Both the annual and diurnal amplitudes are small. About 3 mb amplitude over the year is recorded in central New Zealand with a maximum in autumn

(Table 3.6.1). This is the same order of magnitude as the semi-annual oscillation which, according to Schwerdtfeger et al. (1956) has maxima close to the equinoxes especially in mid-latitudes. They also cite examples of associated wind speeds which, as already seen, may create a semi-annual variation of precipitation.

The surface zonal wind speed shows a definite increase from easterlies in the north to strong (10 knots, 5 m sec^{-1}) westerlies in the south (Table 3.6.2). The meridional component is from the south over most of the region but reverses over eastern New Zealand as if a trough were created by the western coasts. The annual and the diurnal amplitudes are erratic in magnitude and phase.

3.7 Upper Air Observations

For the analysis of the vertical distribution of elements five stations were available on cards although records exist for Raoul and Chatham Islands. Only one sounding per day is made for the measurement of pressure, temperature and humidity at 0000 GMT, noon New Zealand time. Summaries of radiosonde data for the period 1956-1961 have been published by the New Zealand Meteorological Service (1963) and are here compared with the present calculations.

3.7.1 Temperature

Means at the five stations at all 10 levels are generally slightly lower than the average six-year values (Table 3.7.1.1, Fig. 3.7.1.1). However, the differences

TABLE 3.6.1

DETERMINISTIC COMPONENTS OF SURFACE PRESSURE (MILLIBARS)

Station	No.Obs.	Mean	Diurn. Amp.	Ann. Amp.	Date of Max.
Cape Reinga	1436	1015.7	0.17	2.87	Feb.14
Kaitaia	1442	1015.7	0.25	2.99	Feb.11
Mokohinau	1426	1016.1	0.31	3.25	Feb. 7
Whenuapai	1452	1015.4	0.23	3.22	Feb.12
Tauranga	1439	1015.5	0.81	3.16	Feb.13
Gisborne	1436	1015.7	0.34	3.35	Feb.13
New Plymouth	1451	1014.7	0.29	3.45	Feb.18
Ohakea	1448	1014.9	0.34	3.22	Feb.20
Wellington	1455	1014.8	0.33	3.15	Feb.23
Westport	1428	1013.7	0.21	3.72	Feb.21
Cape Farewell	1432	1014.0	0.34	3.12	Feb.27
Nelson	1446	1013.9	0.25	3.19	Feb.28
Christchurch	1445	1012.6	0.24	3.02	Mar.14
Puysegur Pt.	1432	1012.1	0.16	3.74	Mar. 8
Invercargill	1448	1011.7	0.23	3.20	Mar.14
Dunedin	1445	1012.1	0.24	2.98	Mar.12
Campbell Is.	1436	1004.2	0.27	1.09	Mar.13
Chatham Is.	1436	1012.7	0.29	3.83	Jan.21
Lord Howe Is.	1402	1014.8	0.18	0.89	Mar.16
Norfolk Is.	1433	1015.0	0.13	0.23	Jan. 2
Raoul	1441	1015.8	0.07	0.34	Nov.30
Nandi	1427	1011.6	0.47	2.08	Aug. 7

TABLE 3.6.2

DETERMINISTIC COMPONENTS OF SURFACE WIND (KNOTS)

		U Component				V Component			
Station	No.Obs.	Mean	Diurn Amp.	Ann. Amp.	Date of Max.	Mean	Diurn. Amp.	Ann. Amp.	Date of Max.
Cape Reinga	1438	0.3	0.5	3.7	Aug.12	1.5	0.7	1.5	July 12
Kaitaia	1446	0.5	1.6	1.6	Aug.28	0.4	0.9	1.2	May 3
Mokohinau	1430	1.0	0.5	2.4	Aug.20	-0.5	0.1	1.3	Oct.22
Whenuapai	1455	0.9	0.7	0.5	Sept.12	0.0	0.1	1.0	May 3
Tauranga	1499	1.6	1.0	0.1	Dec.28	-0.2	1.2	1.7	June 5
Gisborne	1437	1.7	0.8	1.8	July 16	-0.9	1.6	0.1	Mar. 7
New Plymouth	1456	0.9	2.0	1.8	Jan. 9	1.1	0.8	1.9	June 4
Ohakea	1452	1.3	2.8	1.8	Dec.27	-1.6	2.0	0.2	Nov. 3
Wellington	1455	2.1	0.4	0.7	Dec. 4	-1.3	0.4	2.2	June 12
Westport	1432	1.7	3.0	2.5	Dec.29	2.0	0.8	0.8	Mar.31
Cape Farewell	1433	3.7	2.1	3.3	Dec.24	-0.1	0.6	1.7	Nov. 2
Nelson	1449	0.4	0.6	0.7	Apr.18	-1.0	2.6	2.4	June 8
Christchurch	1448	-0.5	1.8	2.1	June 22	-0.2	0.5	0.9	June 6
Puysegur Pt.	1438	6.7	1.6	3.8	Jan.26	-7.4	0.5	3.1	May 29
Invercargill	1453	3.9	1.6	2.7	Feb. 4	-0.0	2.1	1.3	Dec.25
Dunedin	1450	3.1	1.3	1.3	Nov. 2	-0.2	0.9	1.0	Jan.20
Campbell Is.	1443	11.1	0.6	2.2	Mar.20	-1.4	0.3	3.3	June 22
Chatham Is.	1438	4.1	0.4	2.3	Aug. 6	0.2	0.4	2.8	June 22
Lord Howe Is.	1406	-2.5	0.6	5.0	Aug.17	0.6	0.2	2.1	May 30
Norfolk Is.	1438	-1.9	0.5	5.9	Aug.16	0.2	0.6	0.2	July 23
Raoul Is.	1446	-2.4	0.7	7.5	Feb. 1	1.1	0.4	1.1	Nov. 4
Nandi	1445	0.3	2.8	0.7	Dec.19	0.2	1.3	1.2	Aug. 1

are small and in most cases are less than 0.5°C. Only in 3 cases is the difference greater than one degree. There is the usual decrease of temperature with height at a rate slightly less in magnitude than the wet adiabatic and the usual decrease with increasing latitude.

The annual amplitude shows two maxima (Fig. 3.7.1.2) one at the surface and one between 300 and 400 mb with a minimum at 900 mb. There is no uniform latitudinal variation although Nandi, as might be expected from its tropical location, experiences very little annual change. The date of the maximum lags with height (Fig. 3.7.1.3) especially in the lower 200 mb, to the 300 mb level, above which the temperature trend is reversed. Nandi, with a small amplitude at all levels, has a greatly varying phase angle.

Heights of the pressure surfaces are closely associated with temperature and follow the same pattern (Table 3.7.1.2).

3.7.2 Moisture

Moisture analysis similarly is in agreement with the six-year calculations. There is a marked decrease in the mean mixing ratio with height in sympathy with the temperature but whilst Nandi stands out with a maximum at all levels the other four stations record almost identical amounts (Table 3.7.2.1, Fig. 3.7.2.1). The same applies to the annual amplitude except that, at Nandi between 850 and 800 mb, and at Campbell Island between 850 and 700 mb, the normal decrease with height

TABLE 3.7.1.1

DETERMINISTIC COMPONENTS OF TEMPERATURE (°C)

Station	Pressure Level	Sfc.	900	850	800	700	600	500	400	300	200
Nandi	6 Yr.Mean	28.1	19.9	16.7	14.7	9.8	2.9	-5.4	-16.1	-31.2	-53.1
	Mean	28.1	19.6	16.6	14.4	9.6	2.0	-6.5	-17.1	-32.0	-54.2
	Ann.Amp.	1.1	1.4	1.2	0.6	0.1	0.2	0.7	0.9	0.7	0.6
	Date	Jan.27	Feb.8	Feb.25	Feb.26	Nov.13	Dec.3	E3d.29	Jan.12	Jan.4	Oct.23
Whenuapai	6 Yr.Mean	17.4	9.2	6.6	4.5	-0.6	-7.8	-17.1	-29.0	-43.6	-53.7
	Mean	17.0	8.6	5.9	3.9	-1.0	-8.1	-17.4	-29.3	-43.5	-54.2
	Ann.Amp.	4.4	3.1	3.5	4.1	4.8	5.2	5.7	5.9	4.6	3.6
	Date	Jan.21	Jan.26	Jan.28	Feb.2	Jan.31	Jan.31	Jan.30	Jan.31	Feb.2	Aug. 6
Christchurch	6 Yr.Mean	14.2	8.4	5.7	2.9	-3.0	-10.3	-19.8	-31.7	-46.4	-54.9
	Mean	14.3	7.6	5.0	2.4	-3.3	-10.6	-19.9	-31.8	-46.4	-54.8
	Ann.Amp.	6.8	4.7	4.7	4.8	5.1	5.3	5.6	6.1	5.6	2.4
	Date	Jan.6	Jan.15	Jan.20	Jan.24	Jan.25	Jan.28	Jan.28	Jan.26	Jan.24	Spt. 3
Invercargill	6 Yr.Mean	12.0	5.9	3.2	0.8	-4.7	-11.8	-21.2	-33.0	-47.3	-55.4
	Mean	11.4	5.2	2.6	0.2	-5.1	-12.2	-21.4	-33.1	-47.3	-54.6
	Ann.Amp.	5.6	3.6	3.6	3.8	4.3	5.1	5.5	6.0	5.9	1.6
	Date	Jan.2	Jan.14	Jan.21	Jan.22	Jan.23	Jan.25	Jan.24	Jan.24	Jan.23	Oct.11
Campbell Is.	6 Yr.Mean	7.7	1.5	-0.5	-2.1	-7.6	-14.8	-23.7	-36.2	-48.9	-55.0
	Mean	7.9	1.3	-0.5	-2.4	-8.1	-14.9	-24.5	-35.9	-48.8	-54.0
	Ann.Amp.	2.7	2.5	3.0	3.3	3.7	4.0	4.7	4.9	5.5	3.3
	Date	Jan.10	Jan.21	Jan.20	Jan.22	Jan.18	Jan.26	Jan.21	Jan.23	Jan.27	Dec.28

TABLE 3.7.2.1.

DETERMINISTIC COMPONENTS OF MIXING RATIO (g/kg.)

Station		Sfc.	900	850	800	700	600	500	400	300
Nandi	Mean	15.7	11.7	10.0	8.0	4.9	3.2	1.9	0.9	0.3
	Ann.Amp.	2.2	1.5	1.3	1.5	1.3	0.7	0.5	0.2	0.0
	Date	Mar.2	Mar.17	Mar.11	Mar.15	Feb.23	Feb.23	Feb.21	Jan.31	
Whenuapai	Mean	8.6	5.9	4.9	3.4	2.0	1.3	0.7	0.4	
	Ann.Amp.	1.5	1.2	1.0	0.5	0.5	0.4	0.3	0.2	
	Date	Jan.30	Jan.25	Feb.5	Feb.15	Feb.1	Jan.31	Feb.5	Feb.7.	
Christchurch	Mean	6.9	4.1	3.5	2.9	1.8	1.2	0.6	0.3	
	Ann.Amp.	1.8	1.0	0.9	0.7	0.5	0.4	0.3	0.2	
	Date	Jan.18	Jan.29	Jan.24	Jan.24	Feb.3	Feb.1	Jan.28	Feb.3	
Invercargill	Mean	6.4	4.1	3.4	2.8	1.7	1.1	0.6	0.3	
	Ann.Amp.	1.5	1.0	0.9	0.7	0.5	0.3	0.2	0.1	
	Date	Jan.8	Jan.20	Jan.20	Jan.24	Jan.27	Jan.23	Jan.8	Jan.24	
Campbell Is.	Mean	5.4	4.0	3.1	2.5	1.8	1.0	0.5	0.2	
	Ann.Amp.	0.9	0.7	0.7	0.7	0.9	0.4	0.2	0.1	
	Date	Dec.31	Jan.21	Jan.29	Feb.2	Dec.24	Jan.19	Jan.31	Feb.4	

TABLE 3.7.1.2

DETERMINISTIC COMPONENTS OF HEIGHT OF STANDARD PRESSURE SURFACES (METRES)

Station		1000	900	850	800	700	600	500	400	300	200
Nandi	Mean	98	1030	1521	2037	3156	4418	5837	7577	9675	12406
	Ann.Amp.	2.4	16	14	12	11	10	7	2	6	11
	Date	Jul.29	Jul.23	Jul.18	Jul.11	Jul.5	Jul.3	Jul.1	May 17	Jan.31	Jan.7
Whenuapai	Mean	126	1021	1492	1987	3063	4278	5655	7303	9298	11947
	Ann.Amp.	3.3	40	46	53	70	93	150	161	208	220
	Date	Feb.15	Feb.6	Feb.5	Feb.4	Feb.4	Feb.3	Jan.28	Feb.1	Feb.1	Feb.2
Christchurch	Mean	99	989	1458	1950	3019	4222	5600	7217	9190	11811
	Ann.Amp.	2.7	34	41	50	68	93	122	159	210	245
	Date	Mar.20	Feb.18	Feb.12	Feb.8	Feb.4	Feb.2	Feb.1	Jan.31	Jan.30	Jan.27
Invercargill	Mean	102	972	1436	1925	2985	4181	5551	7159	9121	11737
	Ann.Amp.	3.2	35	40	47	62	83	111	149	201	239
	Date	Mar.19	Feb.22	Feb.16	Feb.14	Feb.7	Feb.4	Feb.1	Jan.31	Jan.29	Jan.26
Campbell Is.	Mean	92	894	1354	1836	2887	4058	5423	7011	8959	11831
	Ann.Amp.	1.2	15	20	28	40	57	82	114	162	757
	Date	Mar.18	Feb.12	Feb.12	Feb.5	Jan.31	Feb.20	Jan.25	Jan.25	Jan.27	Jan.22

MEAN TEMPERATURE

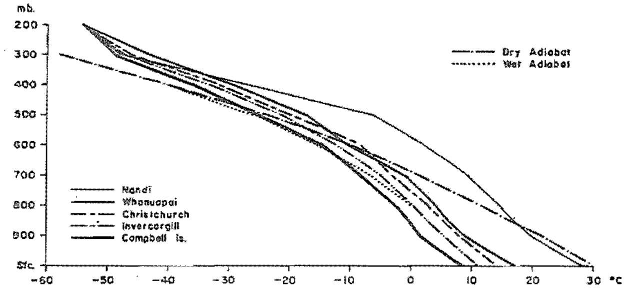


Fig. 3.7.1.1

MEAN MIXING RATIO

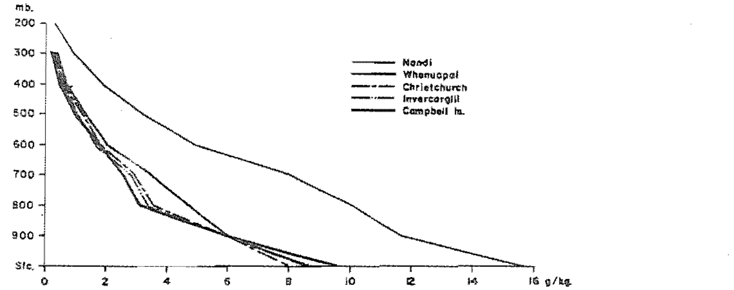


Fig. 3.7.2.1

ANNUAL AMPLITUDE OF TEMPERATURE

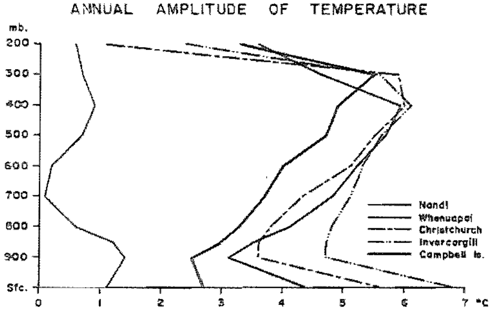


Fig. 3.7.1.2

ANNUAL AMPLITUDE OF MIXING RATIO

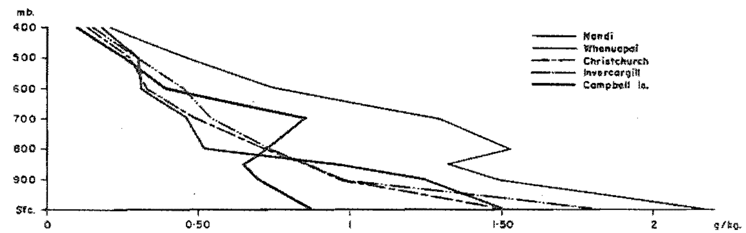


Fig. 3.7.2.2

PHASE OF ANNUAL AMPLITUDE OF TEMPERATURE

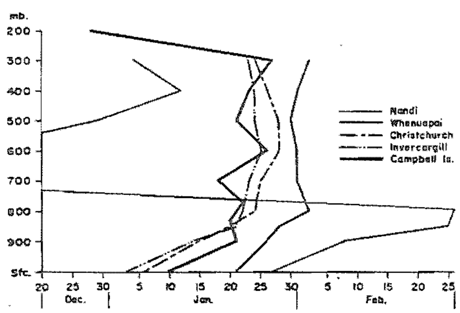


Fig. 3.7.1.3

PHASE OF ANNUAL AMPLITUDE OF MIXING RATIO

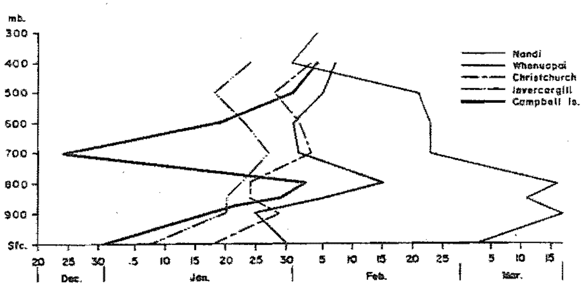


Fig. 3.7.2.3

is reversed (Fig. 3.7.2.2).

Like the temperature, maximum mixing ratio values are recorded in late January and early February except at Nandi where they occur in March and late February (Fig. 3.7.2.3).

3.7.3 Zonal Component of Wind

The zonal wind shows a general increase with height to a maximum at 200 mb (Table 3.7.3.1, Fig. 3.7.3.1). Great speeds in the low and middle troposphere occur at the two southern stations whilst an easterly component is evident below 750 mb at Nandi. In the high troposphere two maxima are recorded; one to the north and one to the south.

Maximum values of the annual amplitude are found between 300 and 400 mb except at Whenuapai where they occur at 200 mb or higher (Fig. 3.7.3.2). At the lower levels the phase angle is erratic but above 800 mb two regimes are clearly defined (Fig. 3.7.3.3). Both Whenuapai and Nandi experience maximum zonal winds in winter whilst at the three southern stations the late summer season is favoured. This is in good agreement with New Zealand Meteorological Service calculations.

3.7.4 Meridional Component of Wind

The mean meridional component is small reaching an average value of -1.5 m sec^{-1} above the surface at Campbell Island and Invercargill (Table 3.7.4.1, Fig. 3.7.4.1). At Nandi it also reaches -1.5 m sec^{-1} in the mid-troposphere but it becomes positive at high levels. Again these follow the longer term values. However,

TABLE 3.7.3.1.

DETERMINISTIC COMPONENTS OF ZONAL WIND (m/sec.)

Station		Sfc.	900	800	700	600	500	400	300	200
	Mean	0.6	-1.7	-0.2	2.2	4.9	7.1	10.7	15.4	19.3
Nandi	Ann.Amp.	1.4	0.7	0.9	1.9	3.8	4.4	5.9	8.0	7.9
	Date	Dec.10	Jan.14	Jly.23	Spt.15	Aug.23	Aug.25	Aug.27	Aug.26	Aug.30
	Mean	0.7	2.8	4.3	5.4	6.6	9.0	12.0	15.6	20.9
Whenuapai	Ann.Amp.	0.5	1.5	1.8	2.7	2.8	2.8	3.8	5.7	8.5
	Date	Aug.29	Aug.30	Jly.22	Jly.17	Jly.22	Jly.14	Jly.16	Jly.5	Jly.17
	Mean	0.1	1.5	4.8	5.9	7.7	9.6	11.8	14.5	15.9
Christchurch	Ann.Amp.	1.1	0.2	1.3	1.6	1.8	2.4	2.4	2.1	2.4
	Date	Jun.27	May 31	Jan.5	Jan.27	Feb.10	Feb.26	Feb.24	Mar.6	May 17
	Mean	2.5	6.2	8.9	9.7	11.1	12.0	13.9	15.4	16.8
Invercargill	Ann.Amp.	1.6	2.0	2.5	2.9	4.1	4.3	4.8	5.3	3.0
	Date	Jan.18	Feb.23	Jan.28	Jan.27	Jan.30	Feb.8	Feb.4	Jan.22	Jan.26
	Mean	5.9	11.9	13.5	14.1	15.9	17.5	20.4	22.8	22.8
Campbell Is.	Ann.Amp.	1.0	0.9	2.0	2.0	2.9	2.7	3.1	3.0	1.3
	Date	Apr.2	Mar.14	Feb.28	Feb.25	Mar.5	Feb.18	Feb.20	Feb.8	Feb.6

TABLE 3.7.4.1

DETERMINISTIC COMPONENTS OF MERIDIONAL WIND (m/sec.)

Station		Sfc.	900	800	700	600	500	400	300	200
	Mean	-0.1	-0.8	-1.6	-1.8	-1.9	-1.9	-1.3	-0.1	0.7
Nandi	Ann.Amp.	0.8	1.0	0.6	0.3	0.8	1.3	1.5	1.2	1.5
	Date	Jly.8	Spt.13	Oct.29	Nov.20	Dec.31	Jan.19	Feb.14	Mar.4	Jan.19
	Mean	-0.1	-0.6	-0.1	0.1	-0.2	-0.2	-0.8	-1.4	-0.6
Whenuapai	Ann.Amp.	0.4	1.1	1.3	1.5	1.5	1.6	1.5	2.3	4.1
	Date	Apr.21	Mar.22	Apr.25	Mar.25	Mar.29	Mar.20	Mar.9	Feb.20	Jly.25
	Mean	0.2	-0.5	-0.5	0.2	0.6	0.3	-0.5	-0.5	-0.3
Christchurch	Ann.Amp.	0.5	1.0	1.5	1.5	1.3	1.4	1.3	1.4	2.3
	Date	May 17	Jun.23	May 24	May 22	May 24	May 11	May 21	Mar.4	Mar.2
	Mean	-0.6	-1.7	-2.0	-1.0	-1.1	-1.4	-2.1	-2.5	-1.4
Invercargill	Ann.Amp.	0.8	0.8	1.4	1.7	1.6	1.4	1.2	1.6	0.8
	Date	Dec.24	May 25	May 31	May 11	May 23	Jun.15	Jun.3	Jun.6	Mar.9
	Mean	-0.6	-2.1	-1.4	-1.3	-1.8	-1.6	-1.9	-1.2	-1.7
Campbell Is.	Ann.Amp.	1.9	1.7	1.4	1.8	2.1	2.2	2.3	2.6	2.5
	Date	Jun.21	Jun.30	Jun.12	Jun.20	Jun.13	May 30	May 17	Apr.21	May 15

the means for Whenuapai and Christchurch are generally small and negative whilst the Meteorological Service figures are positive. The annual amplitudes are generally between 1 and 2 m sec^{-1} with the date of maximum falling in autumn except for Nandi where the amplitude is small (Figs. 3.7.4.2. and 3.7.4.3).

3.8 Conclusion

The present chapter has been almost wholly descriptive. The mean values have been given with little attempt at explanation, for whilst, from a statistical viewpoint, they are the base levels for the discussion of variability they are also the end products of all disturbances. Consequently, any explanation must refer to the intensity and frequency of the fluctuations themselves.

This only partly applies to the two most outstanding frequency components, the annual and diurnal cycles. They may be related, if only indirectly, to the earth's planetary motions and their explanation, therefore, is apparently straightforward. The same cannot be said of the other truly periodic components which come from the fitting of Fourier series to monthly normal data. These components may be useful in description but they appear to be of little value in explanation.

With this initial discussion of deterministic components it is now possible to move on to the more fundamental elements, the frequencies produced by weather, as revealed by the variance spectra.

MEAN ZONAL WIND

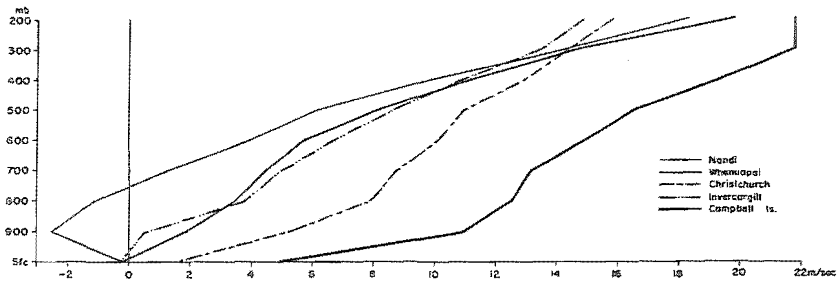


Fig. 3.7.3.1

MEAN MERIDIONAL WIND

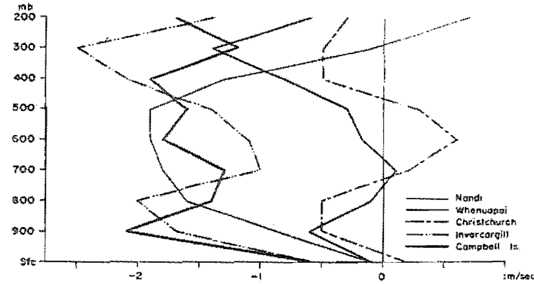


Fig. 3.7.4.1

ANNUAL AMPLITUDE OF ZONAL WIND

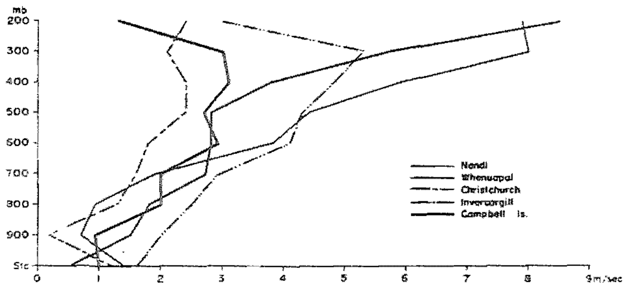


Fig. 3.7.3.2

ANNUAL AMPLITUDE OF MERIDIONAL WIND

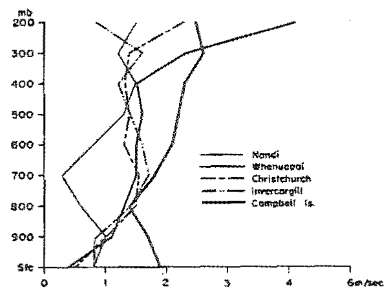


Fig. 3.7.4.2

PHASE OF ANNUAL AMPLITUDE OF ZONAL WIND

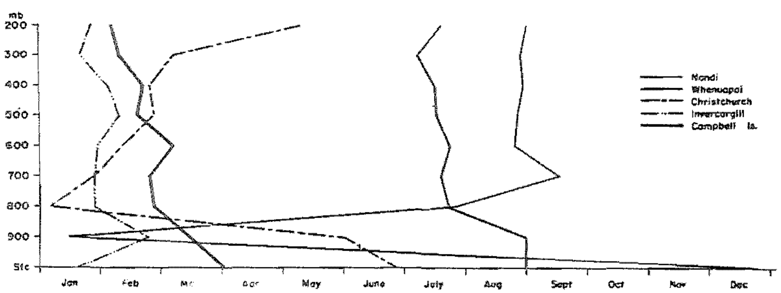


Fig. 3.7.3.3

PHASE OF ANNUAL AMPLITUDE OF MERIDIONAL WIND

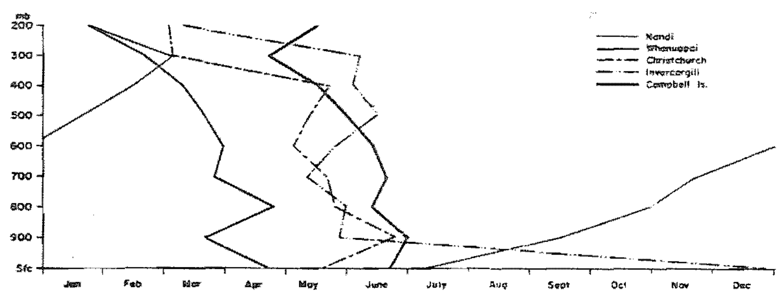


Fig. 3.7.4.3

CHAPTER 4

VARIANCE SPECTRA

4.1 Introduction

Before the results of the calculations on single time series are presented it is convenient to consider briefly what relationships exist between the time and spatial spectra. The local time variation of an atmospheric quantity, f , may be expressed as

$$\frac{\partial f}{\partial t} = - \mathbb{C} \cdot \nabla f + \frac{df}{dt} \qquad 4.1\ (1)$$

where $\mathbb{C}$ is the translation vector, ∇f is the 3 dimensional distribution of f, and $\frac{df}{dt}$ is the time change of f following a particle. Here the local change may be thought of as being due to (a) the movement of a static spatial pattern over the station and (b) the deformation or evolution of that pattern.

For a general zonal flow two cases may be considered. In the first if there is no development, or it is small relative to $\mathbb{C} \cdot \nabla f$, the time spectrum is related to the space spectrum by $\mathbb{C}$. If $\mathbb{C}$ is small for all wavelengths the spectrum will be concentrated in the low frequencies regardless of wavelength. If $\mathbb{C}$ increases with wavelength, as is the normal rule, the forms of the two spectra will be similar (Fig. 4.1.1). In the second case if $\mathbb{C}$ is zero, there will be a relationship between the spectra only if the rate of development is directly

RELATIONSHIPS BETWEEN WAVELENGTH, WAVESPEED AND FREQUENCY

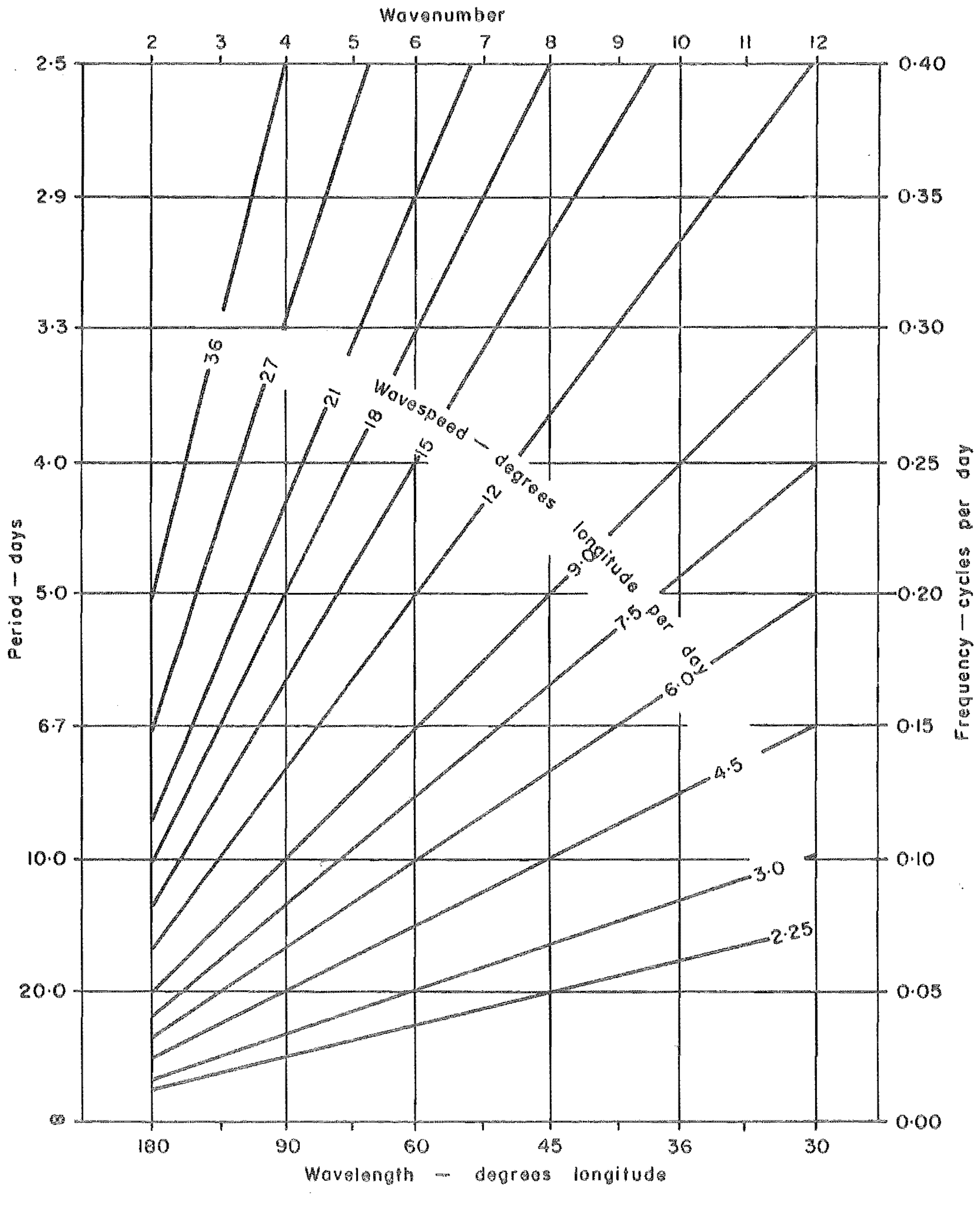


Fig. 4.1.1

proportional to wavelength and occurs uniformly in all areas. In practice these two cases occur together so no detailed correspondence should be expected.

Although confidence intervals were included in the output of all spectra in the present calculations, they have been omitted from the diagrams. It was felt that the significance of the various peaks and valleys is better judged by a comparison between different series, since interest here is centred on a general regional pattern rather than on an individual set of data. Moreover, as pointed out in Chapter 2 the confidence intervals are unreliable because of non-stationarity. Smoothing was carried out on the six-hourly synoptic observations of pressure, temperature, dew point and wind, but not on the other series. For the five upper air stations the temperature spectra were calculated for the 10 standard levels, including the surface, to 200 mb. The mixing ratio was negligible at the four southern stations above 400 mb and at Nandi above 300 mb so here the number of levels was reduced to 7 and 8 respectively. For wind the number was 9 since data for 850 mb were not available on cards. Above 200 mb the number of missing observations increased to such an extent that the spectra were considered unreliable and have been omitted.

4.2. Surface Pressure

Over the annual period of August 1962 to August 1963 the total variance of pressure shows a uniform increase from north to south across the area. Minimum variation of 8.7 mb^2 occurs at Nandi which lies just to the north of the mean position of the Subtropical High. It is typical of tropical regions where the vertical vector of the earth's rotation is small and the pressure disturbances are rapidly damped by the air flow. Maximum variation of 130 mb^2 develops at Campbell Island on the edge of the circumpolar low pressure zone, the region of maximum cyclonic activity (Karelsky 1959). Between these extremes there is an almost linear change and deviations from the pattern are small (Fig. 4.2.1).

The remarkable feature of the spectra is the concentration in the lower frequencies. This is apparent at all stations. Over 95% of the total variance falls in frequencies smaller than 0.4 cycles per day. With the exception of Nandi this includes only 5-9% accounted for by the annual oscillation. At Nandi, although the magnitude of the variance of 1 cycle per year is relatively small, 2.2 mb^2 compared with 5.0 mb^2 at Wellington, it accounts for 25% of the total variance. The diurnal and other tidal frequencies appear to be relatively unimportant although the semi-annual oscillation reaches a peak in central New Zealand.

Peaks occur at 0.0250, 0.0625, 0.1250 and 0.2000 cycles per day in nearly all data with the frequency at 0.0625 cycles per day dominating. However, the importance of the peaks varies depending upon whether interest is centred on frequency or period. In this case the period of 0.0625 cycles per day (16 days) dominates from whichever angle the spectrum is viewed, except in the

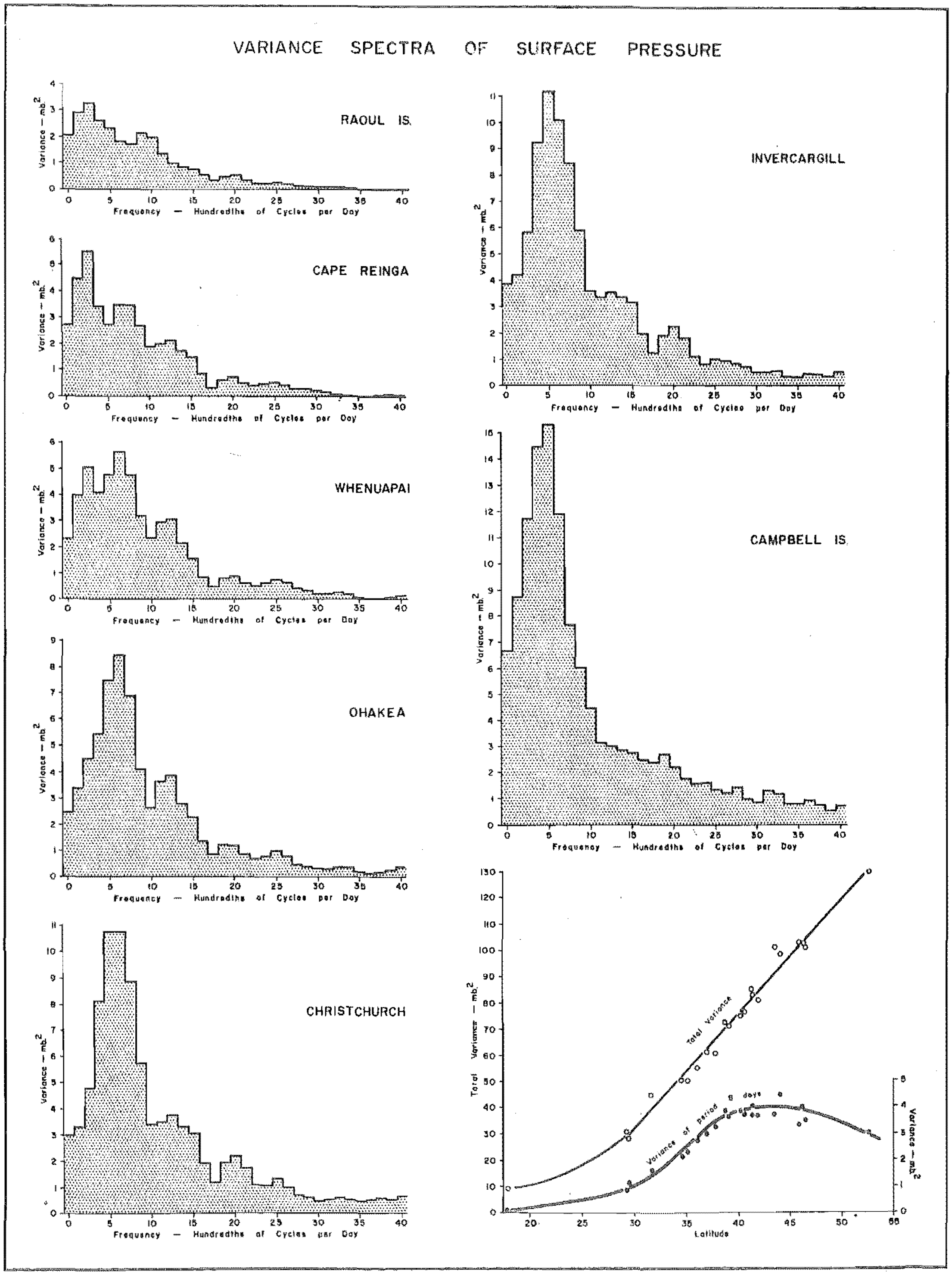

VARIANCE SPECTRA OF SURFACE PRESSURE
RAOUL IS.
CAPE REINGA
WHENUAPAI
OHAKEA
CHRISTCHURCH
INVERCARGILL
CAMPBELL IS.
Variance — mb^2
Frequency — Hundredths of Cycles per Day
Total Variance — mb^2
Total Variance
Variance of period 8 days
Variance — mb^2
Latitude

Fig. 4.2.1

northern stations where on a frequency scale 0.0250 cycles per day is larger.

In general the relative magnitudes of the separate bands remain constant as shown by Table 4.2.1, but the actual magnitudes increase southwards in proportion to the total variance. At the period of 8 days there is a tendency for a maximum in central and northern New Zealand, but this is not significant when adjacent frequencies are considered.

These time fluctuations in pressure, as shown by equation 4.1 (1) are related to the intensity, size, speed of movement and rate of development of the pressure variations in the spatial domain. Emphasis is usually placed upon the scale of cyclones and anticyclones which have a period of between 5 and 9 days (0.2000 to 0.1125 cycles per day), the time of passage of fronts and closed isobars. Indeed 20% of the total variance of pressure falls in this band, yet the present calculations reveal that longer period fluctuations are as, if not more, important. Such frequencies are undoubtedly masked by the detailed mapping of the pressure field. That this is so in the Northern Hemisphere was shown long ago by Rossby et al. (1939), who used maps of data averaged over several days.

4.3 Surface Temperature

In contrast to pressure, total temperature variance shows no consistent latitudinal variation (Table 4.3.1). The subtropical stations record a relatively

TABLE 4.2.1

VARIANCE OF SURFACE PRESSURE

Station	No.of Obs.	Total Var. (mb)^2	% of Total Variance in the Indicated Frequency Groups						
					Cycles per day				
			Diurn.	Ann.	< 0·4	> 0·4	< 0.2375	< 0.1875	< 0.1125
Cape Reinga	1436	50.78	0	8	89	2	82	77	60
Kaitaia	1442	50.39	0	9	90	1	84	79	62
Mokohinau	1426	55.85	0	10	89	1	82	78	60
Whenuapai	1452	61.59	0	8	88	4	82	77	59
Tauranga	1439	61.37	0	8	90	2	82	78	59
Gisborne	1436	72.76	0	8	92	0	82	77	58
New Plymouth	1451	71.38	0	8	90	2	83	78	59
Ohakea	1448	75.84	0	7	92	1	83	79	60
Wellington	1455	85.53	0	6	89	5	80	75	57
Westport	1428	81.61	0	9	89	2	82	76	58
Cape Farewell	1432	76.73	0	6	90	4	83	78	59
Nelson	1446	83.83	0	6	90	4	82	77	57
Christchurch	1445	101.65	0	5	90	5	81	74	58
Puysegur Pt.	1432	102.31	0	7	92	1	84	77	59
Invercargill	1448	101.31	0	5	93	2	85	78	62
Dunedin	1445	103.04	0	4	92	4	84	76	60
Campbell Is.	1436	130.10	0	0	97	3	86	79	67
Chatham Is.	1436	99.07	0	7	92	1	83	78	62
Lord Howe Is.	1402	45.25	0	1	91	8	83	77	62
Norfolk Is.	1433	30.83	0	0	93	7	87	82	65
Raoul Is.	1441	28.34	0	0	92	7	85	79	66
Nandi	1427	8.74	0	25	52	23	49	46	36

TABLE 4.3.1.

VARIANCE OF SURFACE TEMPERATURE

Station	No.of Obs.	Total Var. (°C)^2	% of Total Variance in the Indicated Frequency Groups						
					Cycles per day				
			Diurn	Ann.	< 0.4	> 0.4	< 0.2375	< 0.1875	< 0.1125
Cape Reinga	1439	9.1	10	51	21	18	17	14	11
Kaitaia	1445	16.9	22	38	20	20	16	14	11
Mokohinau	1430	10.4	8	65	17	10	14	13	10
Whenuapai	1455	25.5	25	35	19	21	15	14	11
Tauranga	1089	26.6	32	39	18	9	14	12	9
Gisborne	1434	27.0	23	40	22	15	18	16	11
New Plymouth	1456	21.9	16	39	22	23	18	16	11
Ohakea	1455	24.6	20	44	23	13	18	16	12
Wellington	1457	15.6	10	52	25	13	21	18	12
Westport	1433	21.2	21	41	19	19	17	15	11
Cape Farewell	1434	26.5	17	35	19	29	13	11	7
Nelson	1451	30.2	21	51	15	13	13	12	7
Christchurch	1450	37.1	19	44	19	18	14	13	8
Puysegur Pt.	1439	13.5	4	42	35	19	27	23	14
Invercargill	1453	28.3	16	41	24	21	18	16	10
Dunedin	1450	15.6	7	51	23	19	17	14	9
Campbell Is.	1443	9.5	3	31	39	27	28	24	16
Chatham Is.	1440	12.4	6	46	27	21	20	17	12
Lord Howe Is.	1346	11.0	8	57	18	17	14	12	8
Norfolk Is.	1400	8.6	10	53	18	19	15	14	11
Raoul Is.	1447	9.2	12	37	23	28	19	17	13
Nandi	1434	13.9	55	5	13	27	10	10	8

low variance which is to be expected since the seasonal amplitude of solar radiation is correspondingly small. Further, they are sufficiently far away from the sources of cold air not to experience large advective changes and, as island stations, they are dominated by the influence of the high heat capacity of the water. The latter applies too to the southern oceanic stations. Mainland sites are less influenced by the ocean and radiative factors become important. The largest variance is recorded at Christchurch where normal advective factors are amplified. High temperatures are recorded with northwest fohn winds often just prior to the northward passage of a cold front.

The annual and diurnal components as already discussed account for about 60% of the total variance, although there are exceptions, such as Campbell Island, where these frequencies tally only 34%. The island is subject to high cloudiness so net radiation shows little diurnal amplitude. At all stations except Nandi the annual component is the most important single frequency.

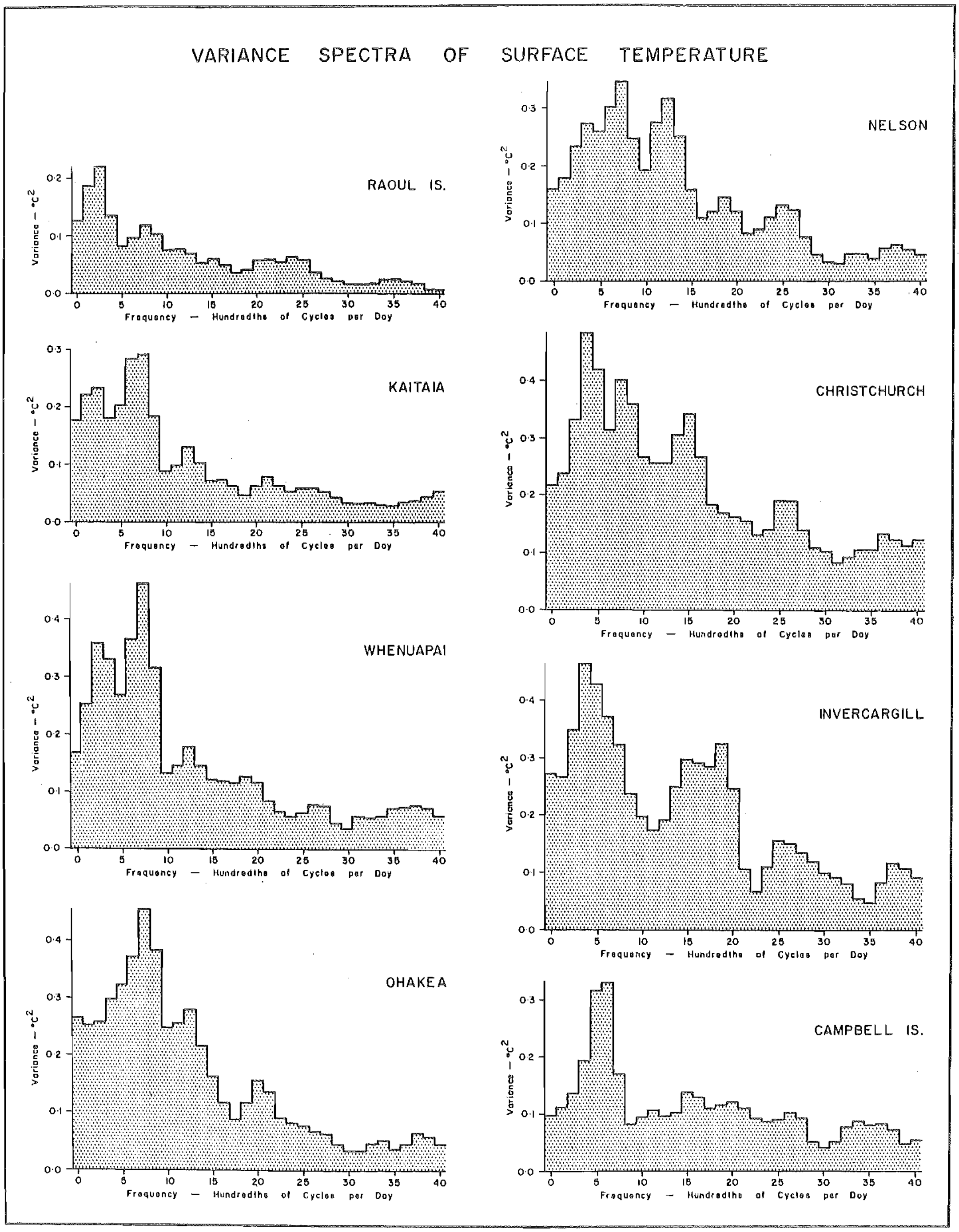
Whilst there is a concentration of variance in the lower frequencies there is no rapid decline to the higher frequencies. In fact about 19% of the total variance, as calculated from six-hourly observations, lies in frequencies greater than 0.4 cycles per day. As can be seen in Fig. 2.5.2 the spectrum in these frequencies is relatively flat, resembling a random series. It is here suggested that part of this variance is noise due to the rounding of the raw data since temperature is given to the nearest whole degree.

Definite peaks occur at similar frequencies as in pressure but they are now more variable (Fig. 4.3.1). In the north the lower frequency of 0.0125 cycles per day dominates followed by minor peaks centred at about 0.0750 and 0.1375 cycles per day. To the south the low frequency peak appears to move through 0.0250 cycles per day to 0.0375 cycles per day although it is there overshadowed by higher frequencies. Over most of the mainland 0.0750 cycles per day becomes dominant but is closely followed by 0.1250 cycles per day. In the south these change to 0.0625 and 0.1500 cycles per day. Also there is a suggestion of a further peak at 0.2000 cycles per day. In general then 26, 15, 7½ and 5 day periods dominate with regional variation especially in the meridional direction.

For temperature equation 4.1 (1) becomes

$$\frac{\partial T}{\partial t} = -\mathbb{V} \cdot \nabla_H T - (\gamma_d - \gamma) w + \frac{1}{c_p} \frac{dH}{dt}, \qquad 4.3\ (1)$$

where the pressure terms have been ignored since they are at least one order of magnitude smaller than the remaining components. The first term on the right hand side, horizontal advection, on the average is less than $\pm$ 10°C per day but in extreme situations such as in Christchurch it may reach −10°C per hour. The second term, adiabatic heating (including latent heat released during condensation), although important in the free atmosphere, is small at the surface. There at the boundary w must be zero. Maximum warming through this



VARIANCE SPECTRA OF SURFACE TEMPERATURE
RAOUL IS.
KAITAIA
WHENUAPAI
OHAKEA
NELSON
CHRISTCHURCH
INVERCARGILL
CAMPBELL IS.
Variance — °C²
Frequency — Hundredths of Cycles per Day

Fig. 4.3.1

process therefore takes place above the surface and often creates an inversion which will in turn inhibit vertical mixing, a necessary process for the warming of the lower layers. The same situation may occur under fohn conditions but here the forces producing descent are much greater and so more effective. Adiabatic cooling for a low lying station must be extremely small and similarly can be neglected.

The last term, diabatic heating, is made up of net radiation and turbulent transfer. For a particular layer near the ground, except in unusual circumstances (Funk 1960) turbulent transfer is more important. However, this in turn depends upon the radiative heating of the surface. In the extreme cases of day and of night these two terms have an average heating effect of $\pm$ 10°C per day (Chapter 3) which is the same order of magnitude as advective changes. Unless there is a close relationship between the advective and diabatic terms the two sets of processes will have different spectra. Moreover, short period fluctuations of the same degree, through for example cloud cover variations, will tend to extend the time spectra into higher frequencies. The non-advective processes may therefore explain some of the differences between the pressure and temperature spectra.

4.4 Solar Radiation

Since total incoming solar radiation is only one of four components of net radiation it can act no more than as a guide to the diabatic heating term. Further, the data are limited to four series which makes generalization difficult.

Total variance is at a maximum at Christchurch and a minimum at Nandi (Table 4.4.1). The difference may be accounted for by the annual harmonic which totals over 50% at the mid-latitude stations as compared with 19% at Nandi. In fact, there is a general increase in the annual percentage southwards and a corresponding decrease in the other components in magnitude as well as in percentage. The general shape of the spectra is similar to temperature except at Invercargill where there is a concentration of power in the higher frequencies. Peaks are erratic and it is difficult to recognise any particularly preferred frequencies as in the other series (Fig. 4.4.1). It is not clear whether this inconsistency is real or whether it is the result of sampling.

TABLE 4.4.1

VARIANCE OF SOLAR RADIATION

Station	No. of Obs.	Total Var. $(\text{gcal/day})^2$	% in Ann. Harm.
Nandi	365	22318	19
Whenuapai	363	32913	55
Christchurch	365	44806	67
Invercargill	361	40405	68

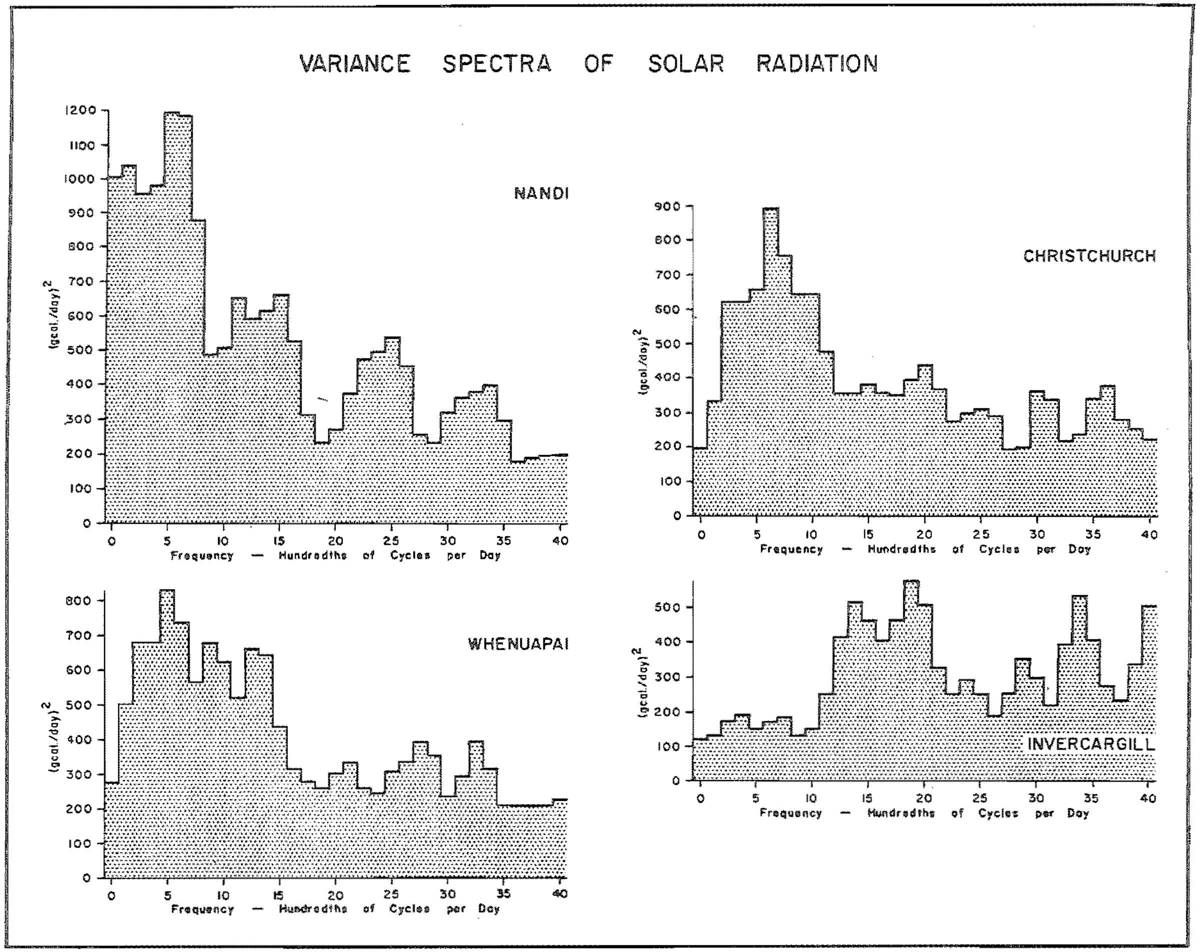

VARIANCE SPECTRA OF SOLAR RADIATION
NANDI
CHRISTCHURCH
WHENUAPAI
INVERCARGILL
(gcal./day)^2
Frequency — Hundredths of Cycles per Day

Fig. 4.4.1

Equation 4.1 (1) cannot be applied directly to insolation but only to factors such as cloud amount and atmospheric composition which affect it. Although the raw data are daily totals they refer only to the daylight hours, which in the extreme case of Invercargill, vary between 8½ and 15¾ hours during the year. The results therefore refer to weighted averages of atmospheric transmission which in large part will be advective in origin.

4.5. Surface Dew Point Temperature

It was explained in Chapter 3 that, as a measure of water vapour variation, the dew point temperature will overemphasize the fluctuations for periods when the total vapour content is relatively low. This means that winter variations will be enhanced. However, if the frequency content is similar there should be no serious error. Indeed, comparison with surface mixing ratio data for upper air stations reveals the same frequency concentrations of variance.

Since the oceans maintain a relatively constant surface air temperature and are an unending source for moisture, the total variations of dew point recorded by open coastal stations is a minimum. More protected stations which may receive air from land as well as oceanic trajectories experience the highest variance (Table 4.5.1). The diurnal harmonic accounts for only 1-2% of this, as compared with the annual harmonic of between 20 and 40%. Unlike the dry bulb temperature more than 40% of the

TABLE 4.5.1

VARIANCE OF SURFACE DEW POINT TEMPERATURE

Station	No.of Obs.	Total Var. $(^{\circ}C)^2$	% of Total Variance in the Indicated Frequency Groups						
					cycles per day				
			Diurn	Ann.	< 0.4	> 0.4	< 0.2375	< 0.1875	< 0.1125
Cape Reinga	1440	12.3	1	42	45	11	36	32	24
Kaitaia	1445	15.6	0	30	51	19	42	37	27
Mokohinau	1428	16.2	0	36	47	17	40	37	26
Whenuapai	1454	19.1	1	28	47	24	40	36	26
Tauranga	1087	22.8	1	33	49	17	40	35	25
Gisborne	1436	16.9	2	42	41	15	34	33	23
New Plymouth	1456	20.3	2	34	46	18	38	33	22
Ohakea	1454	19.1	2	37	42	19	36	32	22
Wellington	1457	16.9	0	39	45	16	37	33	22
Westport	1431	19.7	2	46	40	12	34	31	21
Cape Farewell	1435	20.7	2	31	38	29	30	27	17
Nelson	1451	25.1	2	42	41	15	35	35	21
Christchurch	1448	22.7	2	43	33	22	26	24	17
Puysegur Pt.	1438	13.7	1	28	47	24	39	34	22
Invercargill	1455	17.4	5	39	39	17	32	28	20
Dunedin	1450	18.5	1	40	38	21	30	25	17
Campbell Is.	1442	12.9	0	26	54	20	38	33	22
Chatham Is.	1440	15.9	0	21	52	27	40	35	23
Raoul Is.	1447	14.9	0	30	52	18	46	43	36

variance of the dew point lies in frequencies less than 0.4 cycles. This leaves only about 20% in higher frequencies.

Peaks are distinct and more consistent than those of temperature variance (Fig. 4.5.1). To the north the low frequency of 0.0250 cycles per day dominates along with 0.0750 cycles per day. In central New Zealand the relative magnitudes of these two frequencies are reversed and in the far south 0.0625 and 0.0500 cycles per day replace 0.0750 cycles per day. A third peak at 0.1250 cycles per day is important north of Christchurch but to the south there is another change: this time to 0.1500 cycles per day. A minor peak at 0.2000 cycles per day is present throughout. Together these frequencies, less than 0.2000 cycles per day (including the annual), account for two thirds of the total variance.

The factors directly affecting the moisture content are advection, evaporation and condensation. For land stations evaporation is often limited and condensation in the form of fog or dew is limited in time and quickly reversed. Consequently advection is the most important factor affecting frequencies less than 0.4 cycles per day.

4.6 Precipitation

The total variance of daily precipitation is low, but like the monthly normal values of Chapter 3, it is very variable from site to site (Table 4.6.1). Of the six stations used Campbell Island has the smallest

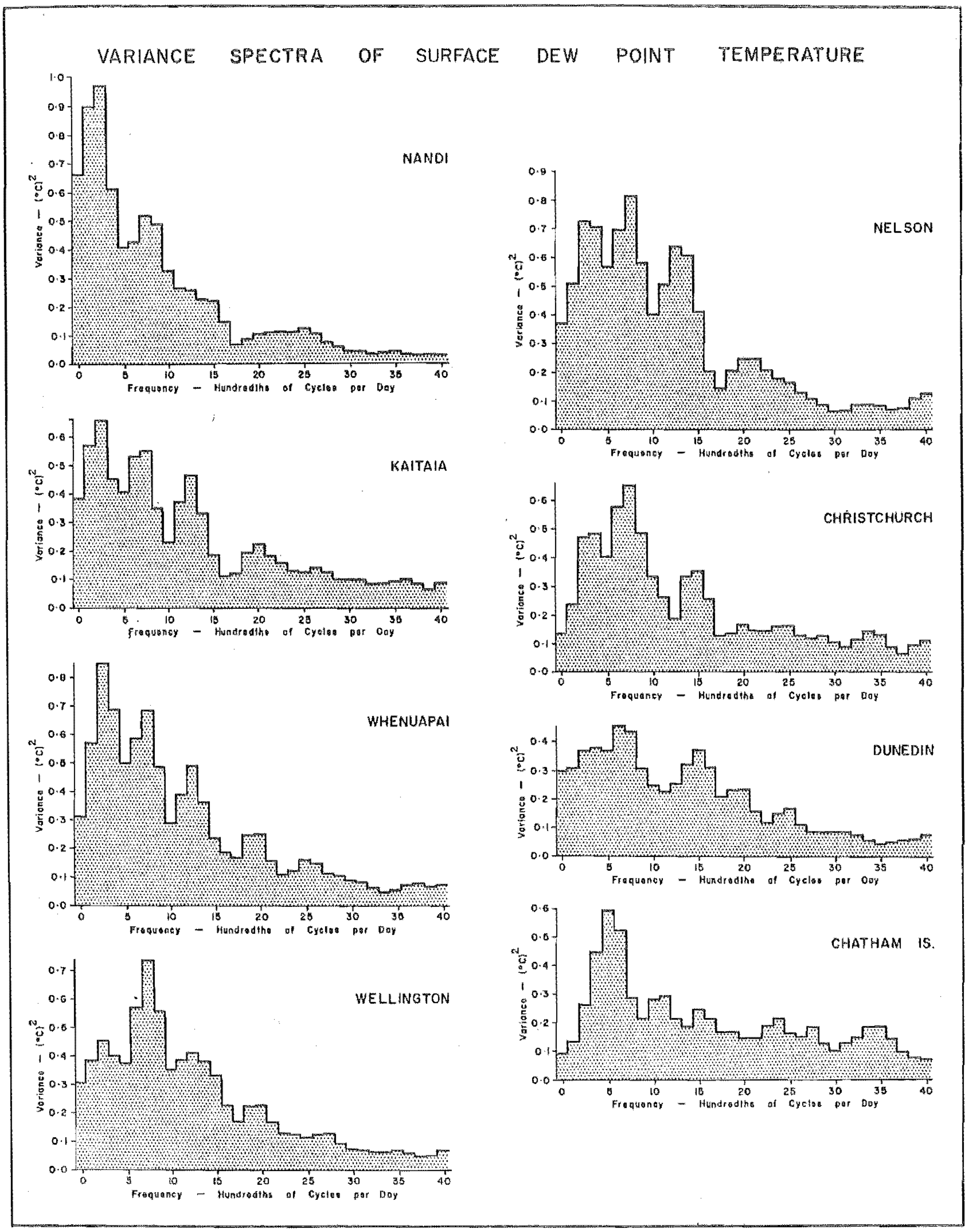

VARIANCE SPECTRA OF SURFACE DEW POINT TEMPERATURE
NANDI
KAITAIA
WHENUAPAI
WELLINGTON
NELSON
CHRISTCHURCH
DUNEDIN
CHATHAM IS.
Variance — (°C)²
Frequency — Hundredths of Cycles per Day

Fig. 4.5.1

variance with 0.023 square inches and Arthurs Pass the largest with 0.673 square inches. The percentage taken up by the annual component is extremely small and amounts to only 4% at Nandi and less than 1% elsewhere.

TABLE 4.6.1

VARIANCE OF PRECIPITATION

Station	No.of Obs.	Total Var. (inches/day)^2	% in Ann. Harm.
Nandi	365	0.302	3
Whenuapai	365	0.121	1
Christchurch	365	0.030	1
Arthurs Pass	365	0.673	0
Invercargill	365	0.044	0
Campbell Is.	365	0.023	0

Like radiation, the precipitation spectra are inconsistent and a large proportion of the variance falls into the higher frequencies (Fig. 4.6.1). Whilst the water vapour content and many of the condensation producing mechanisms must be advective in origin, orographic rain is frequently dependent upon wind speed and direction. These as will be seen in the next section are also very variable in frequency.

4.7 Surface Wind

Whilst calculations were made on the individual meridional and zonal components of wind (Tables 4.7.1

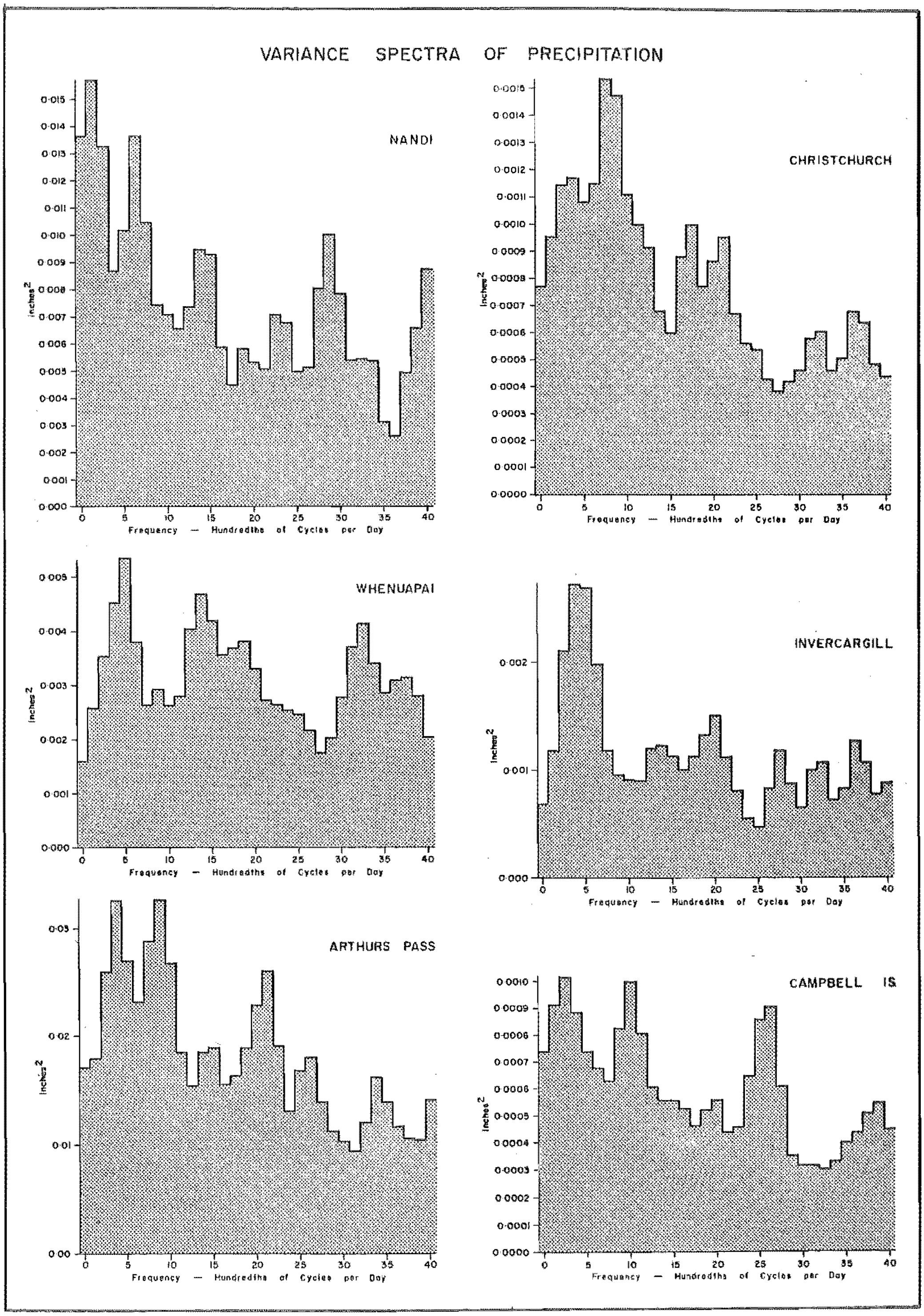

VARIANCE SPECTRA OF PRECIPITATION
NANDI
CHRISTCHURCH
WHENUAPAI
INVERCARGILL
ARTHURS PASS
CAMPBELL IS
Inches²
Frequency — Hundredths of Cycles per Day

Fig. 4.61

and 4.7.2) it is more useful here to look at their variances together. At many sites the direction of the wind is dictated to a large extent by the topography. Hence the individual spectra are somewhat erratic. The average of the individual variance bands of the zonal and meridional components is proportional to the kinetic energy of the wind and, in this case, is less dependent upon direction. These are plotted in Fig. 4.7.1 as knots^2.

Total variance reaches a maximum at exposed sites and there is no apparent regional pattern. Only at the subtropical stations are the periodic components significant and of these Nandi alone has a diurnal oscillation. In most cases 70% of the total variance falls in frequencies less than 0.4 cycles per day. Generally stations with large variance are included in Fig. 4.7.1. This shows that peaks at 0.0250, 0.0625 or 0.1375 and 0.2000 cycles per day appear in the wind data.

4.8 Upper Air Temperature

The annual harmonic dominates the upper air temperature spectra although it declines in importance from the surface to a minimum at 900 mb (Table 4.8.1). Above this level it increases again to 400-300 mb before it shows a definite drop at 200 mb. The remaining variances of temperature reveal different profiles for each station (Fig. 4.8.1). At Nandi the small surface variance of less than $3^{o}C^{2}$ is maintained throughout the troposphere and will be neglected in the following discussion. At Whenuapai the major increase at the lower

TABLE 4.7.1.

VARIANCE OF SURFACE ZONAL WIND COMPONENT

Station	No. of Obs.	Total Var. (knots)^2	% of Total Variance in the Indicated Frequency Groups: Diurn.	Ann.	cycles per day < 0.4	> 0.4	< 0.2375
Cape Reinga	1438	77.8	0	10	65	25	54
Kaitaia	1446	62.2	2	2	77	19	66
Mokohinau	1430	164.5	0	2	83	15	70
Whenuapai	1455	41.6	1	0	78	21	67
Tauranga	1449	40.9	1	0	69	30	57
Gisborne	1437	38.9	1	4	48	47	36
New Plymouth	1456	51.9	4	3	69	24	51
Ohakea	1452	74.1	5	2	64	29	50
Wellington	1455	54.0	0	0	57	43	41
Westport	1432	49.3	9	6	47	38	34
Cape Farewell	1433	123.9	2	4	63	31	51
Nelson	1449	21.3	1	1	47	51	33
Christchurch	1448	47.2	3	4	52	41	35
Puysegur Pt.	1438	178.5	1	4	69	26	57
Invercargill	1453	88.9	2	4	71	23	55
Dunedin	1450	174.7	0	0	74	26	56
Campbell Is.	1443	146.4	0	2	78	20	64
Chatham Is.	1438	127.1	0	2	80	18	64
Lord Howe Is.	1406	82.7	0	15	72	13	64
Norfolk Is.	1438	62.1	0	28	62	10	54
Raoul Is.	1446	145.3	0	19	73	9	65
Nandi	1445	22.3	18	1	29	52	21

TABLE 4.7.2

VARIANCE OF SURFACE MERIDIONAL WIND COMPONENT

Station	No. of Obs.	Total Var. (knots)^2	% of Total Variance in the Indicated Frequency Groups: Diurn.	Ann.	cycles per day < 0.4	> 0.4	< 0.2375
Cape Reinga	1438	249.7	0	0	85	15	69
Kaitaia	1446	46.5	1	2	71	26	59
Mokohinau	1430	161.3	0	1	76	23	57
Whenuapai	1455	34.2	0	1	64	35	49
Tauranga	1449	29.2	2	4	51	43	37
Gisborne	1437	55.3	2	0	57	41	41
New Plymouth	1456	53.1	1	4	70	25	54
Ohakea	1452	36.9	3	0	62	35	50
Wellington	1455	123.2	0	2	70	28	55
Westport	1432	40.3	1	1	48	50	35
Cape Farewell	1433	88.6	0	2	54	44	37
Nelson	1449	43.9	8	6	43	43	30
Christchurch	1448	39.1	0	1	48	51	31
Puysegur Pt.	1438	369.4	0	1	67	31	46
Invercargill	1453	39.2	5	2	42	51	28
Dunedin	1450	150.6	0	0	63	37	42
Campbell Is.	1443	100.2	0	6	63	31	46
Chatham Is.	1438	122.2	0	3	74	23	56
Lord Howe Is.	1406	44.9	0	5	72	23	54
Norfolk Is.	1438	39.5	0	0	84	16	70
Raoul Is.	1446	63.5	0	1	74	25	69
Nandi	1445	16.9	5	4	39	52	28

TABLE 4.8.1

VARIANCE OF UPPER AIR TEMPERATURE

mb	Nandi Total Var. (°C)^2	Nandi % in Ann. Harm.	Whenuapai Total Var. (°C)^2	Whenuapai % in Ann. Harm.	Christchurch Total Var. (°C)^2	Christchurch % in Ann. Harm.	Invercargill Total Var. (°C)^2	Invercargill % in Ann. Harm.	Campbell Is. Total Var. (°C)^2	Campbell Is. % in Ann. Harm.
200	1.7	11	26.4	25	30.2	10	32.5	4	41.1	13
300	2.5	8	20.8	51	24.7	63	27.3	64	24.3	62
400	3.0	13	29.2	60	31.2	59	31.9	56	31.2	39
500	3.0	9	27.1	59	28.5	55	30.2	51	34.6	32
600	2.0	1	22.7	59	26.5	52	26.8	48	44.2	18
700	2.3	0	20.2	57	27.1	48	23.5	40	26.8	26
800	3.0	7	17.0	48	29.6	39	22.3	32	24.3	22
850	3.2	23	13.4	46	29.6	37	21.1	30	20.5	22
900	3.0	31	10.0	49	29.3	38	20.0	33	13.3	24
Sfc.	3.3	18	13.6	71	35.7	65	24.5	64	7.2	52

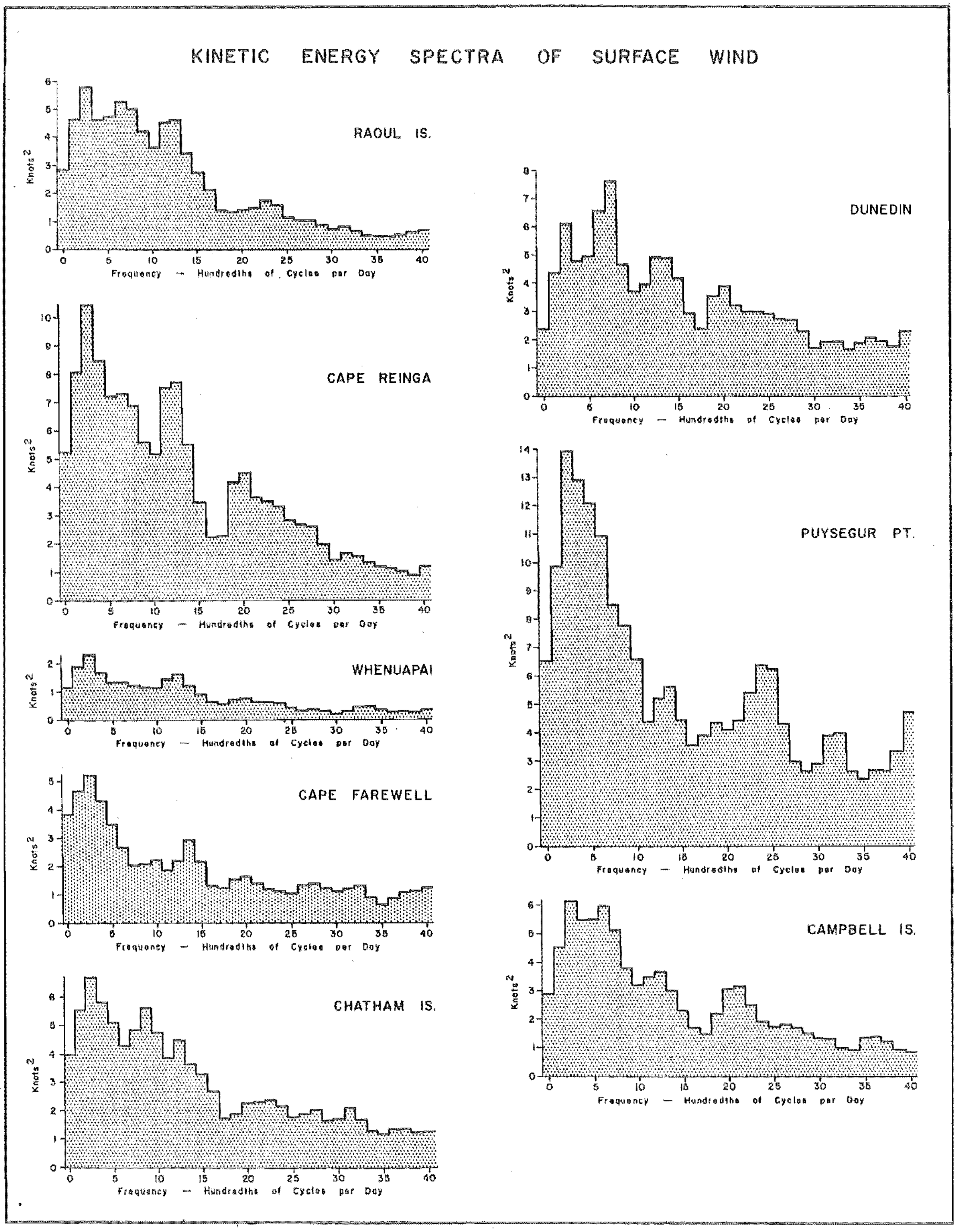

KINETIC ENERGY SPECTRA OF SURFACE WIND
RAOUL IS.
CAPE REINGA
WHENUAPAI
CAPE FAREWELL
CHATHAM IS.
DUNEDIN
PUYSEGUR PT.
CAMPBELL IS.
Knots²
Frequency — Hundredths of Cycles per Day

Fig. 4.7.1

TOTAL VARIANCE OF UPPER AIR TEMPERATURE

(Excluding Annual)

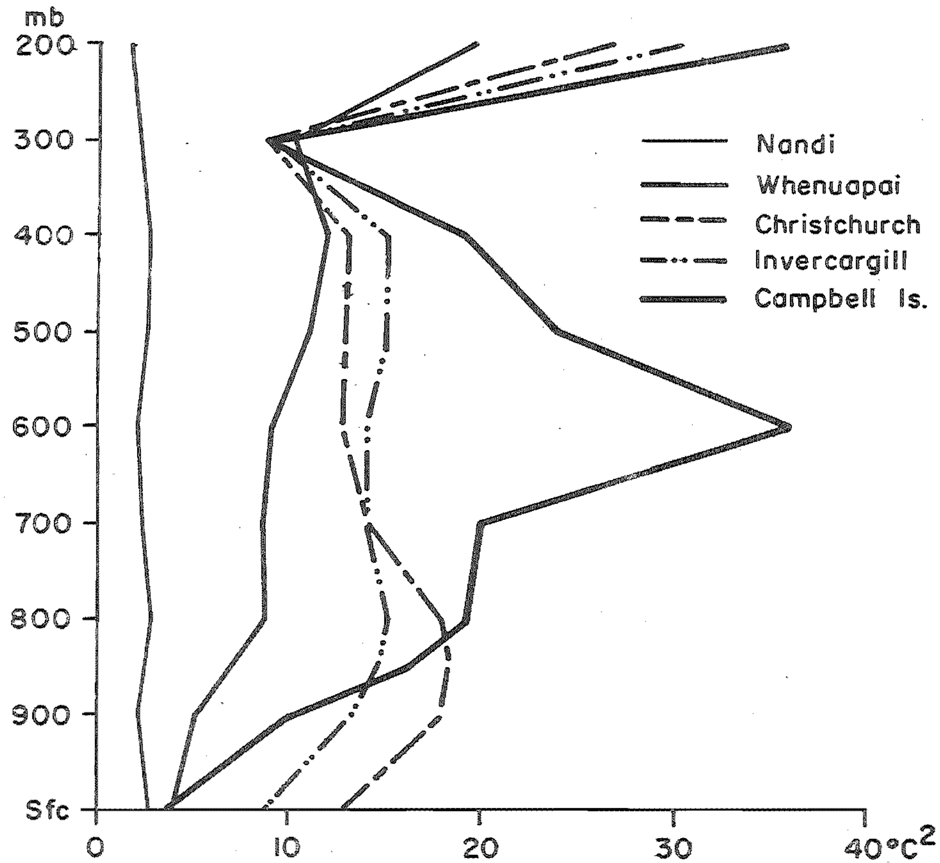


Fig. 4.8.1

levels takes place between 900 and 800 mb, and a maximum is reached at 400 mb with a slight minimum at 300 mb before a marked increase above. The other three stations show this same minimum and sharp increase between 300 and 200 mb. At Christchurch and less so at Invercargill a maximum is recorded between 900 and 800 mb which may be attributed to the influence of the Southern Alps. Campbell Island has an unusual profile showing a distinct peak at 600 mb. These profiles agree well with the New Zealand Meteorological Service (1963) calculations except for the Campbell Island maximum.

The relative importance of the variance spectral bands is remarkably consistent with height at each station, so nothing important is lost through averaging for all levels excluding 850 mb (Fig. 4.8.2). At Whenuapai peaks are present at 0.0250, 0.0625, 0.1250 and 0.2000 cycles per day in that order on a linear frequency scale. The individual levels deviate from this arrangement at 800 mb where 0.0250 and 0.0625 combine into one peak at 0.0500 cycles per day. A peak frequency of 0.0250 cycles per day does not exist at Christchurch, but instead 0.0625 cycles per day dominates above 900 mb followed by 0.1375 and 0.2000 cycles per day. Both 0.0500 and 0.1375 cycles per day appear dominant at the two southern stations with 0.2000 cycles per day at Invercargill becoming less distinct at Campbell Island. A further peak at 0.2375 cycles per day at Campbell Island is also present at upper levels over Invercargill.

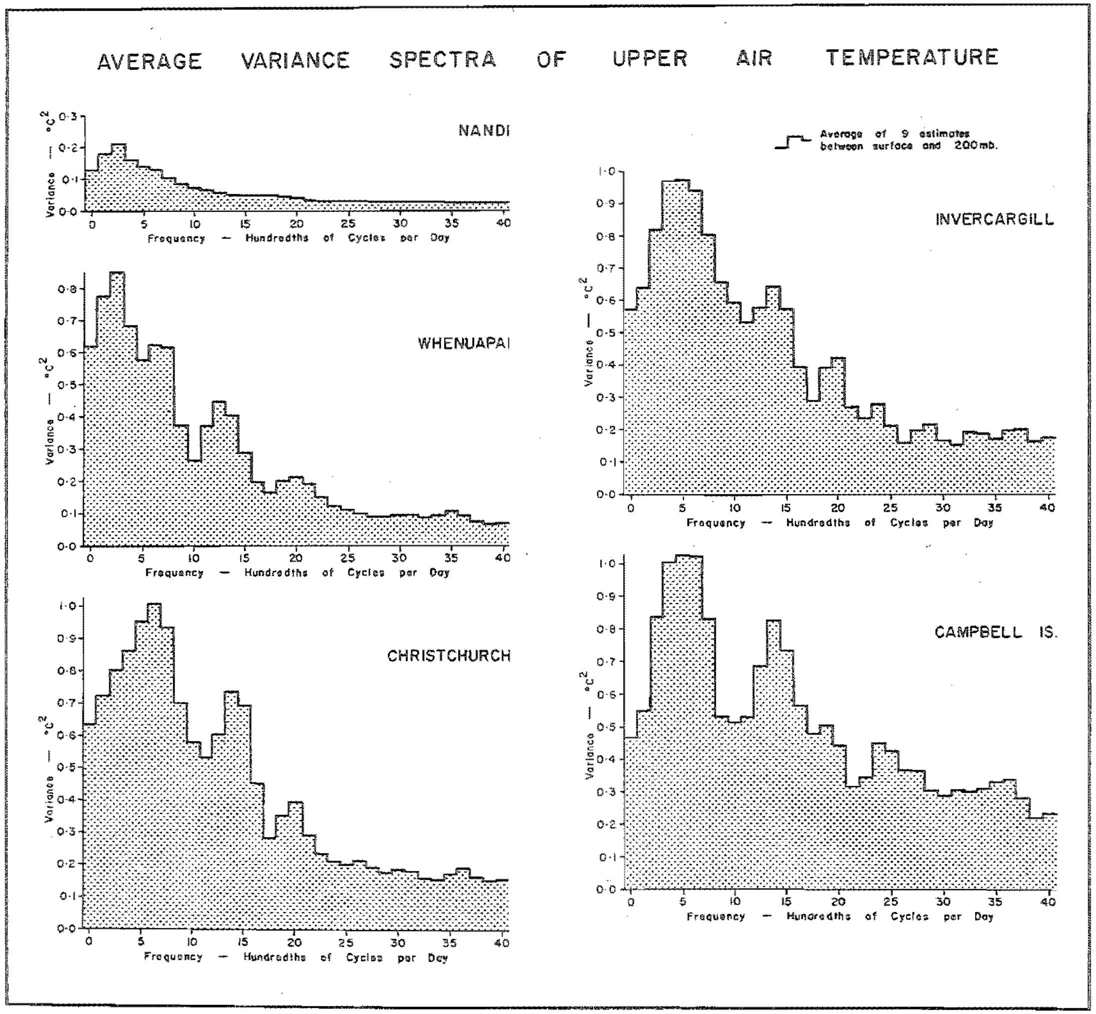

AVERAGE VARIANCE SPECTRA OF UPPER AIR TEMPERATURE
NANDI
WHENUAPAI
CHRISTCHURCH
Average of 9 estimates between surface and 200mb.
INVERCARGILL
CAMPBELL IS.
Variance — °C²
Frequency — Hundredths of Cycles per Day

Fig. 4.8.2

With upper air temperatures the second term in equation 4.3 (1) adiabatic heating, assumes its full importance and often will be the same magnitude as the advective term. For the conversion of potential energy into kinetic energy (Chapter 6) these must have opposite signs so the radiative term will remain significant.

4.9 Upper Air Mixing Ratio

The total variances of mixing ratio, as recorded by the daily midday observations, show a decrease with increasing latitude (Table 4.9.1). Both Nandi and Whenuapai have double maxima: one at the surface and another between 850 and 800 mb, whilst Christchurch and Invercargill have only the one at the surface. Except at the surface less than 20% of this variance is accounted for by the annual harmonic. The remainder are shown in Fig. 4.9.1.

Peaks are less consistent with height compared with temperature although similar frequencies are present (Fig. 4.9.2). The 0.0250 cycles per day band dominates the northern three stations followed by 0.0625 or 0.0750. These are present at the southern stations but less consistently and so appear as 0.0750 and 0.0375 at Invercargill and 0.0625 at Campbell Island. A third peak at 0.1375 or 1500 cycles per day is present in all except Christchurch where 0.1250 is more important. Similarly a further maximum occurs at 0.2000 cycles per day.

TABLE 4.9.1

VARIANCE OF UPPER AIR MIXING RATIO

mb	Nandi Total Var. $(g/kg)^2$	Nandi % in Ann. Harm.	Whenuapai Total Var. $(g/kg)^2$	Whenuapai % in Ann. Harm.	Christchurch Total Var. $(g/kg)^2$	Christchurch % in Ann. Harm.	Invercargill Total Var. $(g/kg)^2$	Invercargill % in Ann. Harm.	Campbell Is. Total Var. $(g/kg)^2$	Campbell Is. % in Ann. Harm.
300	0.01	9								
400	0.22	10	0.07	23	0.04	29	0.04	22	0.02	27
500	1.06	10	0.26	17	0.16	20	0.18	14	0.14	16
600	2.49	10	0.70	11	0.63	16	0.53	10	0.45	17
700	4.88	17	1.58	7	1.19	12	1.17	10	1.34	12
800	7.07	17	2.94	5	1.97	14	1.82	14	1.82	13
850	5.23	17	3.09	15	2.18	17	1.96	19	2.20	10
900	4.22	26	2.78	28	2.40	20	2.05	23	2.00	12
Sfc.	5.88	40	4.09	27	3.74	43	2.90	39	1.73	22

TOTAL VARIANCE OF UPPER AIR MIXING RATIO
(Excluding Annual)

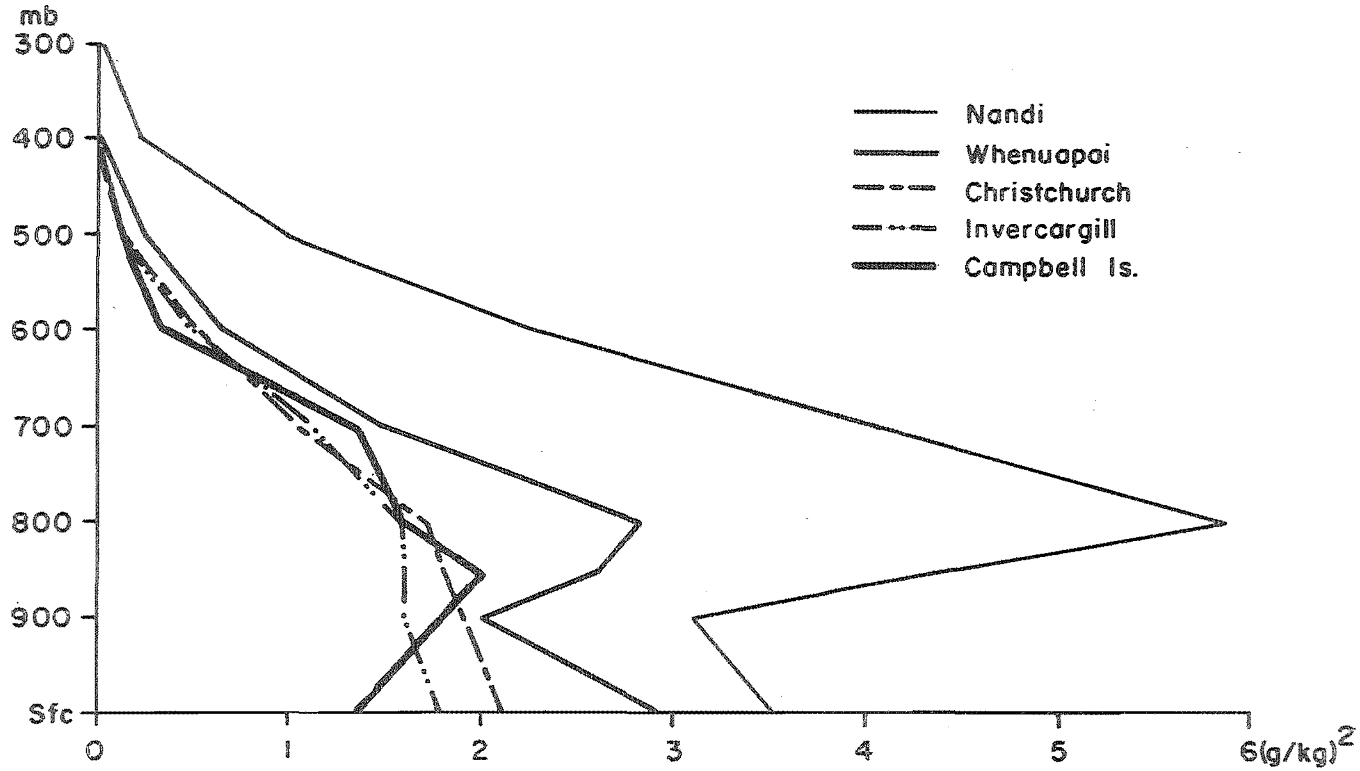
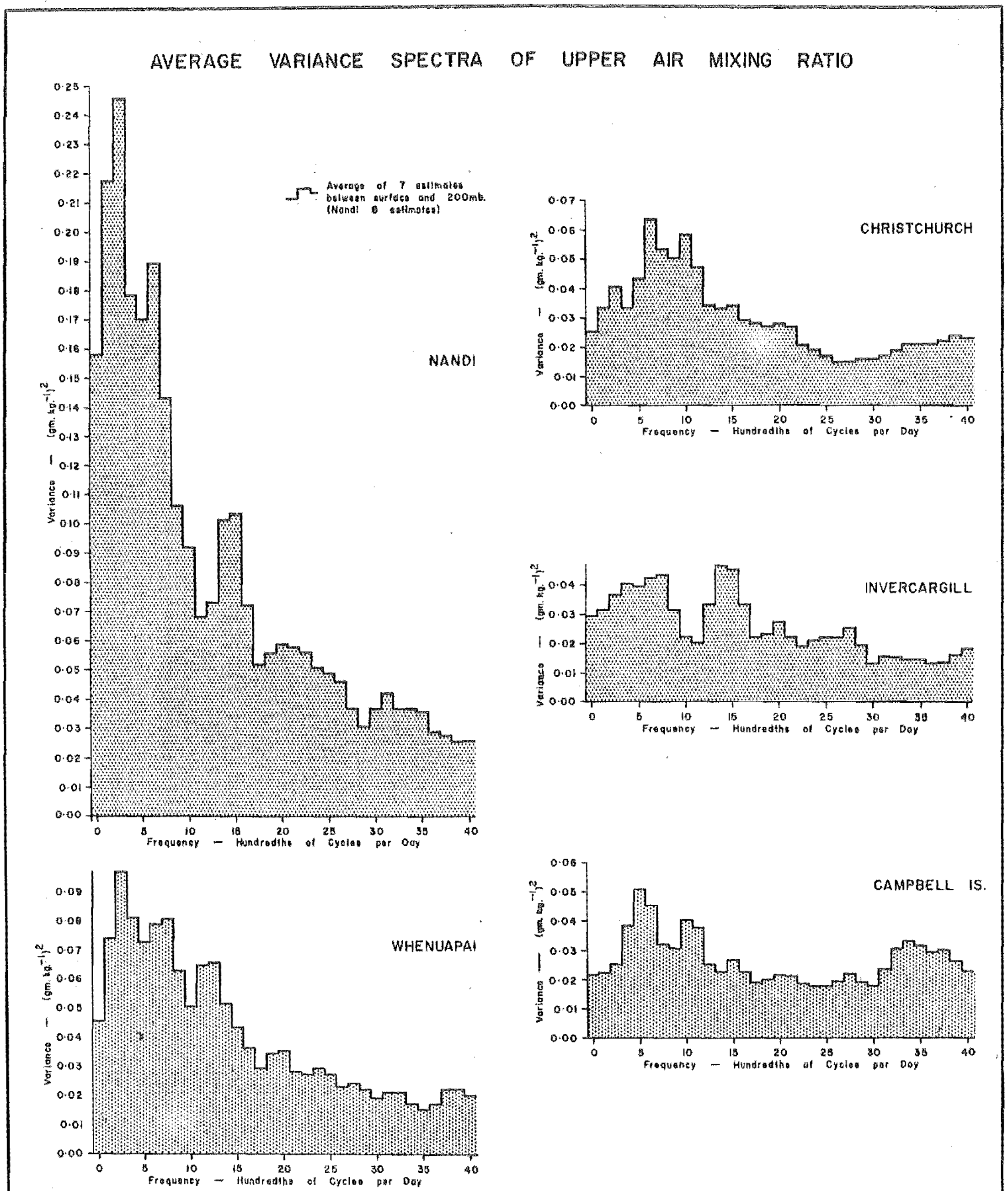


Fig. 4.9.1



AVERAGE VARIANCE SPECTRA OF UPPER AIR MIXING RATIO
Average of 7 estimates between surface and 200mb. (Nandi 6 estimates)
NANDI
CHRISTCHURCH
INVERCARGILL
WHENUAPAI
CAMPBELL IS.
Variance — (gm. kg.$^{-1}$)^2
Frequency — Hundredths of Cycles per Day

Fig. 4.9.2

The breakdown of moisture change reveals three components, horizontal advection, vertical motion which is associated with convergence, and the related condensation and precipitation. It has been assumed that small scale turbulent transfer is relatively unimportant.

4.10 Upper Air Zonal Wind

The variation due to the annual harmonic is small, generally less than 8% except above 600 mb at Nandi. Only at Campbell Island is there a low level maximum to interrupt the gradual increase in total variance with height (Table 4.10.1, Fig. 4.10.1). The three southern stations record maxima at 300 mb but the increase continues to 200 mb at Nandi and Whenuapai.

With this element there is good consistency with height but little with latitude. The major peak falls within the first five frequency bands whilst the second and third peaks at Nandi of 0.1000 and 0.2000 become 0.1375 and 0.1875 cycles per day at Invercargill (Fig. 4.10.2). The 0.1250 band is present below 600 mb but 0.2125 cycles per day is more important at Campbell Island.

These fluctuations in zonal wind may be accounted for by the advected pattern and by the changes in transfer of momentum into the system. Friction will remove westerly momentum at the surface and the eddies will transfer momentum horizontally as well as vertically (see Chapter 6).

TOTAL VARIANCE OF UPPER AIR ZONAL WIND

(Excluding Annual)

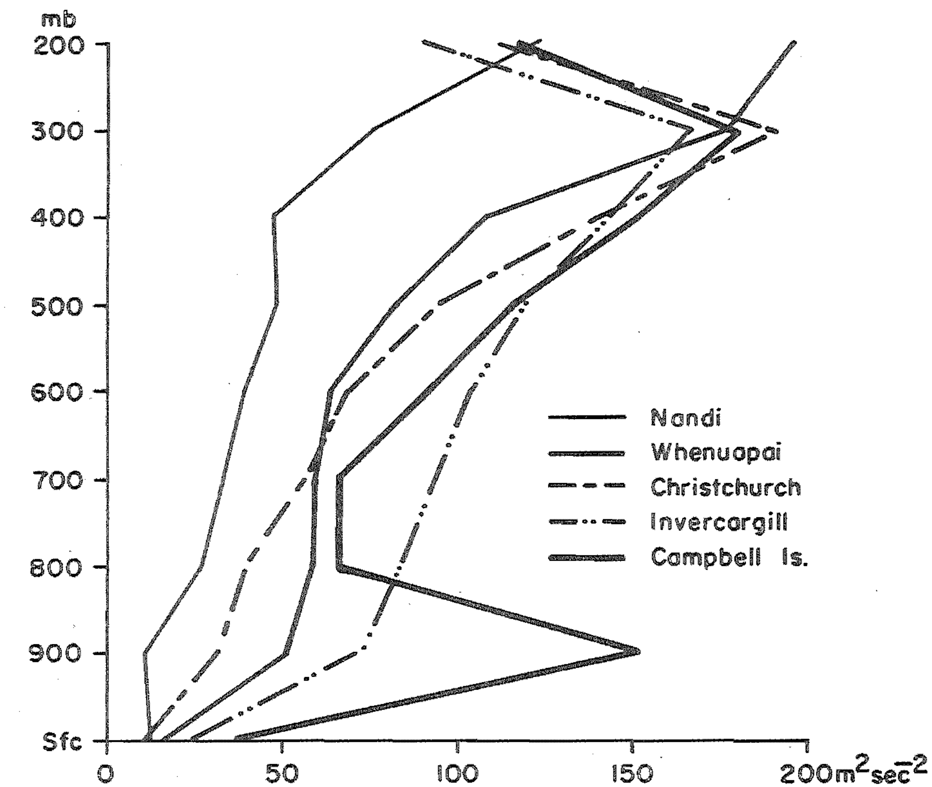


Fig. 4.10.1

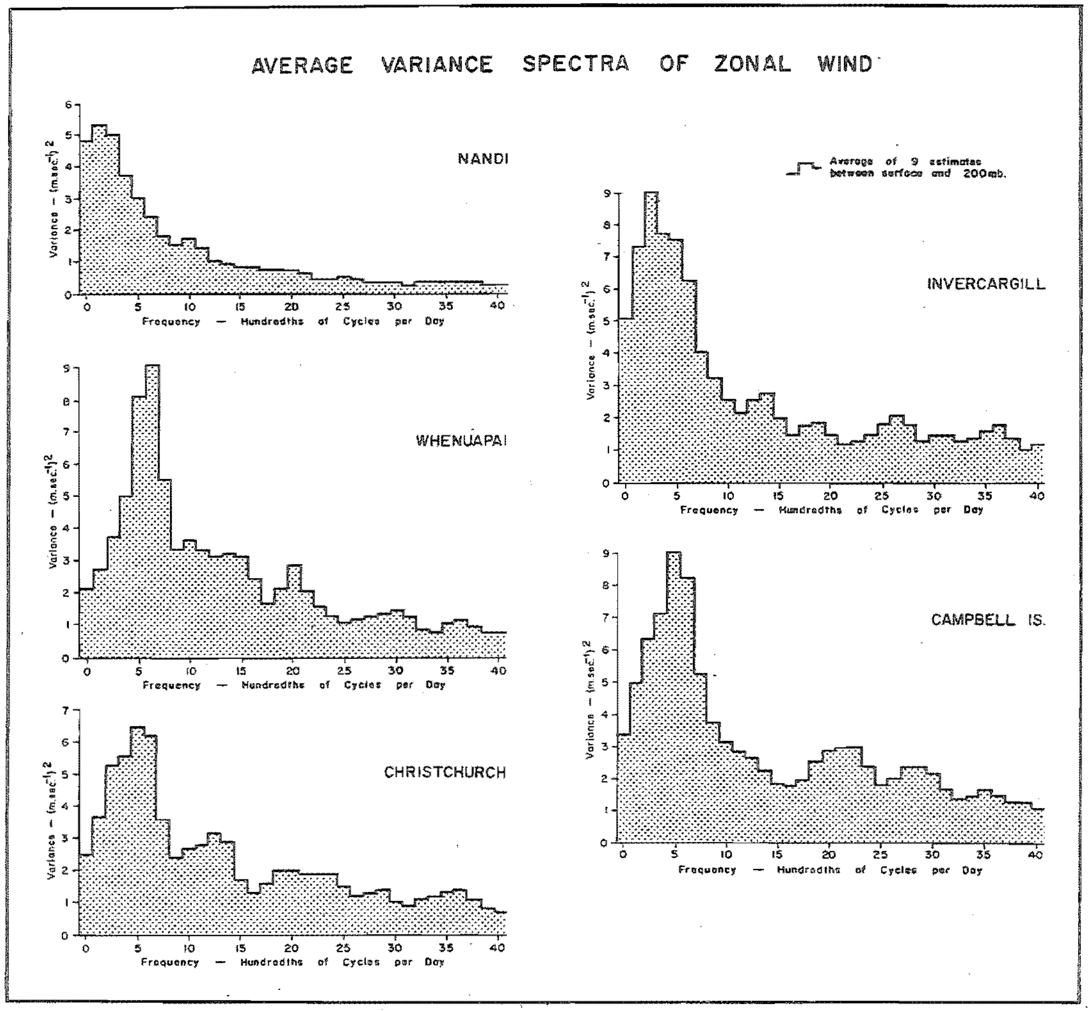

AVERAGE VARIANCE SPECTRA OF ZONAL WIND
NANDI
WHENUAPAI
CHRISTCHURCH
INVERCARGILL
CAMPBELL IS.
Average of 9 estimates between surface and 200mb.
Variance — (m.sec^{-1})^2
Frequency — Hundredths of Cycles per Day

Fig. 4.10.2

4.11 Upper Air Meridional Wind

Relatively the annual harmonic is small and will be neglected in the present discussion (Table 4.11.1). The total variance at all stations increases with height to a maximum at 300 mb (200 mb at Nandi). There is generally an increase with increasing latitude except for Invercargill and Christchurch which are reversed (Fig. 4.11.1). Peaks are relatively consistent and appear at 0.0250, 0.0625-0.0875, 0.1250-0.1275 and 0.1875-0.2000 cycles per day (Fig. 4.11.2).

4.12 Conclusion

This chapter makes it clear that the variance spectra of meteorological elements are continuous and, in agreement with Ward et al. (1961), that only the diurnal and annual components are statistically significant. However, the present study has gone further in estimating the magnitudes and relative importance of these periodic components. Moreover, it has been shown that certain frequencies consistently appear above adjacent ones at different stations and levels, and in different variables. This suggests, despite the lack of any clear-cut statistical significance, that the atmosphere in the New Zealand region during the period under investigation tended to be disturbed in an organised way. Further evidence towards such a conclusion will be brought forward in the next chapter.

TABLE 4.10.1

VARIANCE OF UPPER AIR ZONAL WIND COMPONENT

mb	Nandi Total Var. $(m/sec)^2$	Nandi % in Ann. Harm.	Whenuapai Total Var. $(m/sec)^2$	Whenuapai % in Ann. Harm.	Christchurch Total Var. $(m/sec)^2$	Christchurch % in Ann. Harm.	Invercargill Total Var. $(m/sec)^2$	Invercargill % in Ann. Harm.	Campbell Is. Total Var. $(m/sec)^2$	Campbell Is. % in Ann. Harm.
200	154.6	20	226.5	16	115.5	2	93.2	5	118.4	1
300	107.1	30	193.8	8	193.6	1	177.7	8	184.6	2
400	65.7	27	118.5	6	146.4	2	157.0	7	156.9	3
500	59.0	17	86.5	5	97.3	3	127.7	7	121.5	3
600	47.1	16	68.6	6	71.1	2	112.2	7	95.5	4
700	34.4	5	62.6	6	59.3	2	98.9	4	69.6	3
800	26.3	2	61.1	3	40.5	2	87.8	4	69.6	3
900	9.9	3	53.1	2	31.1	0	75.5	3	151.9	0
Sfc.	13.2	8	18.4	1	13.5	5	26.6	5	38.1	1

TABLE 4.11.1

VARIANCE OF UPPER AIR MERIDIONAL WIND COMPONENT

mb	Nandi Total Var. $(m/sec)^2$	Nandi % in Ann. Harm.	Whenuapai Total Var. $(m/sec)^2$	Whenuapai % in Ann. Harm.	Christchurch Total Var. $(m/sec)^2$	Christchurch % in Ann. Harm.	Invercargill Total Var. $(m/sec)^2$	Invercargill % in Ann. Harm.	Campbell Is. Total Var. $(m/sec)^2$	Campbell Is. % in Ann. Harm.
200	113.3	1	155.6	5	177.2	2	149.5	0	173.4	2
300	87.7	1	195.9	1	269.6	0	255.5	0	318.6	1
400	47.8	3	132.8	1	209.5	0	202.8	4	263.8	1
500	31.6	3	99.8	1	129.0	1	133.1	1	176.2	1
600	28.1	1	77.5	1	92.6	1	92.7	1	129.6	2
700	18.7	0	62.9	2	66.5	2	64.8	2	87.4	2
800	15.7	1	52.4	2	52.1	2	45.6	2	84.1	1
900	11.1	4	40.1	2	46.8	1	33.6	1	84.9	1
Sfc.	10.7	3	13.8	1	12.6	1	11.5	3	25.5	5

TOTAL VARIANCE OF UPPER AIR

MERIDIONAL WIND

(Excluding Annual)

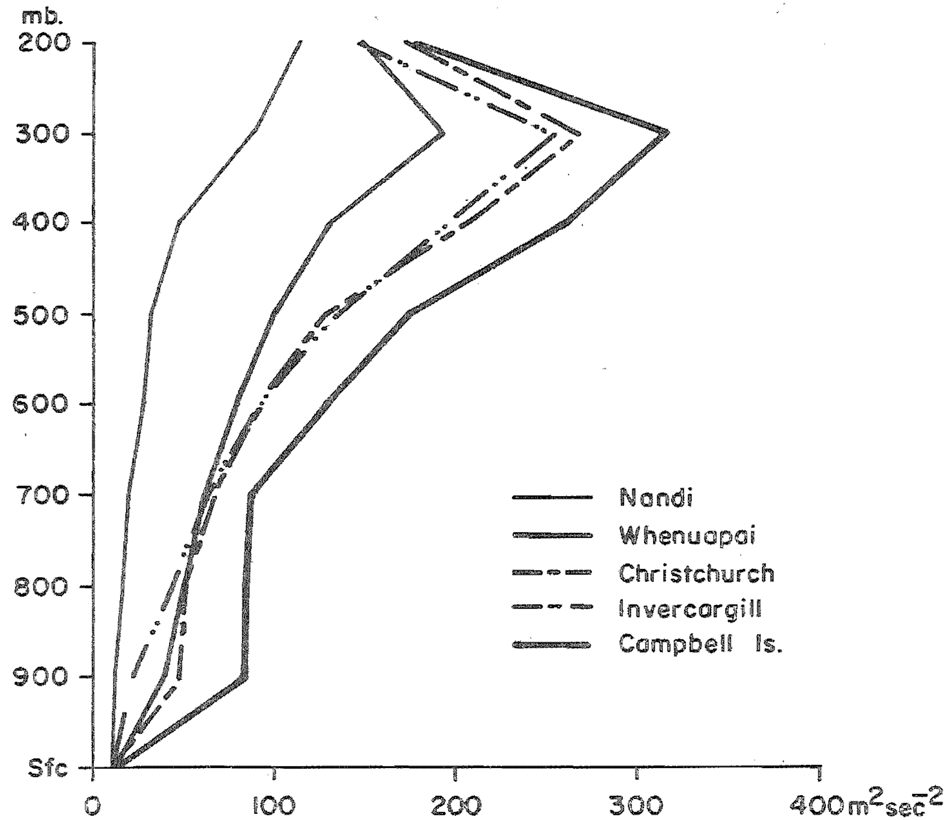


Fig. 4.11.1

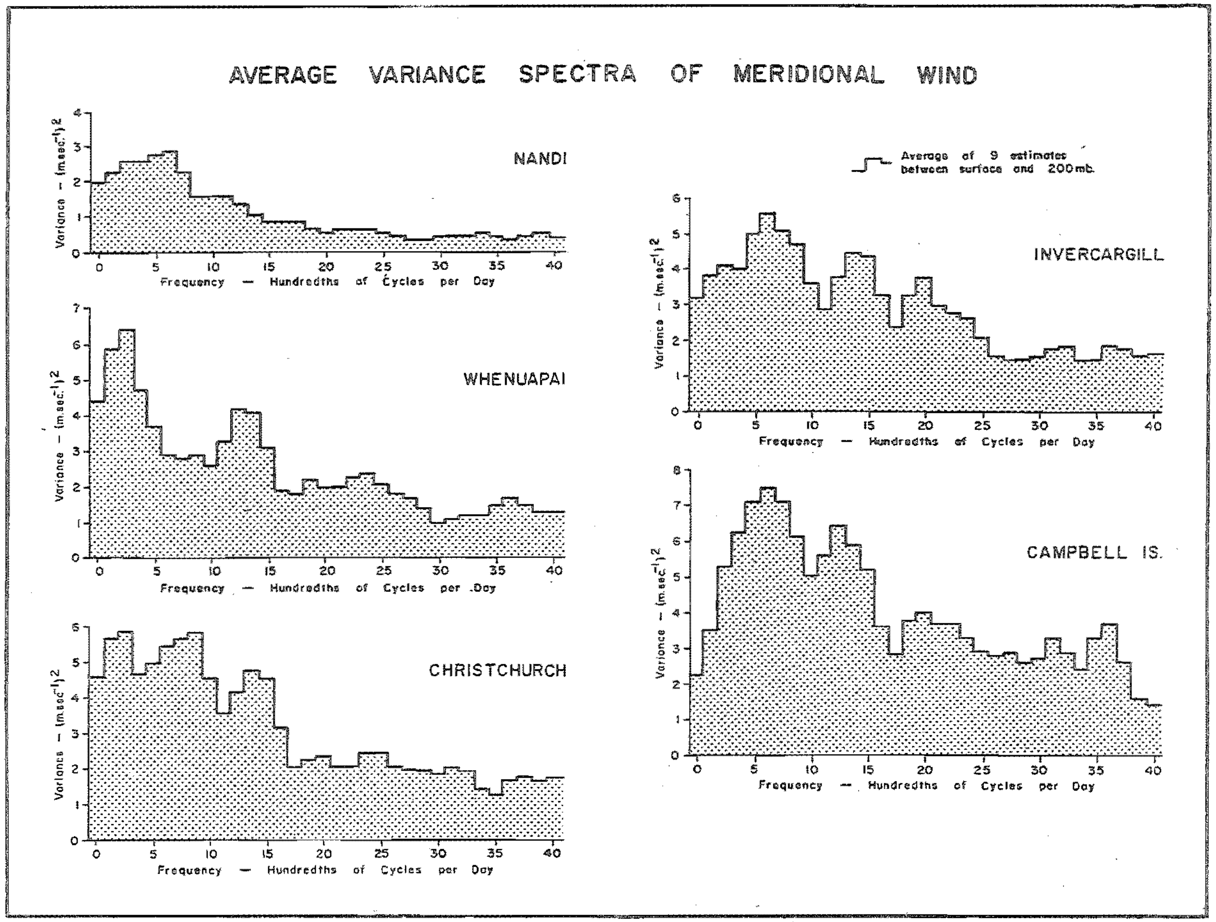

AVERAGE VARIANCE SPECTRA OF MERIDIONAL WIND
NANDI
WHENUAPAI
CHRISTCHURCH
INVERCARGILL
CAMPBELL IS.
Average of 9 estimates between surface and 200mb.
Variance – (m.sec⁻¹)²
Frequency — Hundredths of Cycles per Day

Fig. 4.11.2

CHAPTER 5

CROSS SPECTRA

5.1 Introduction

So far only single series spectra have been analysed. Alone they do not explain or describe climate although they do go some way toward this end. Weather is the result of a complex system which has areal and vertical extent and which is composed of many elements. In this chapter some of the interrelationships in space and between series will be investigated. From the limited information available a simple model will be constructed, which will serve both to describe and in part to explain average systems, and hence climate of the New Zealand region.

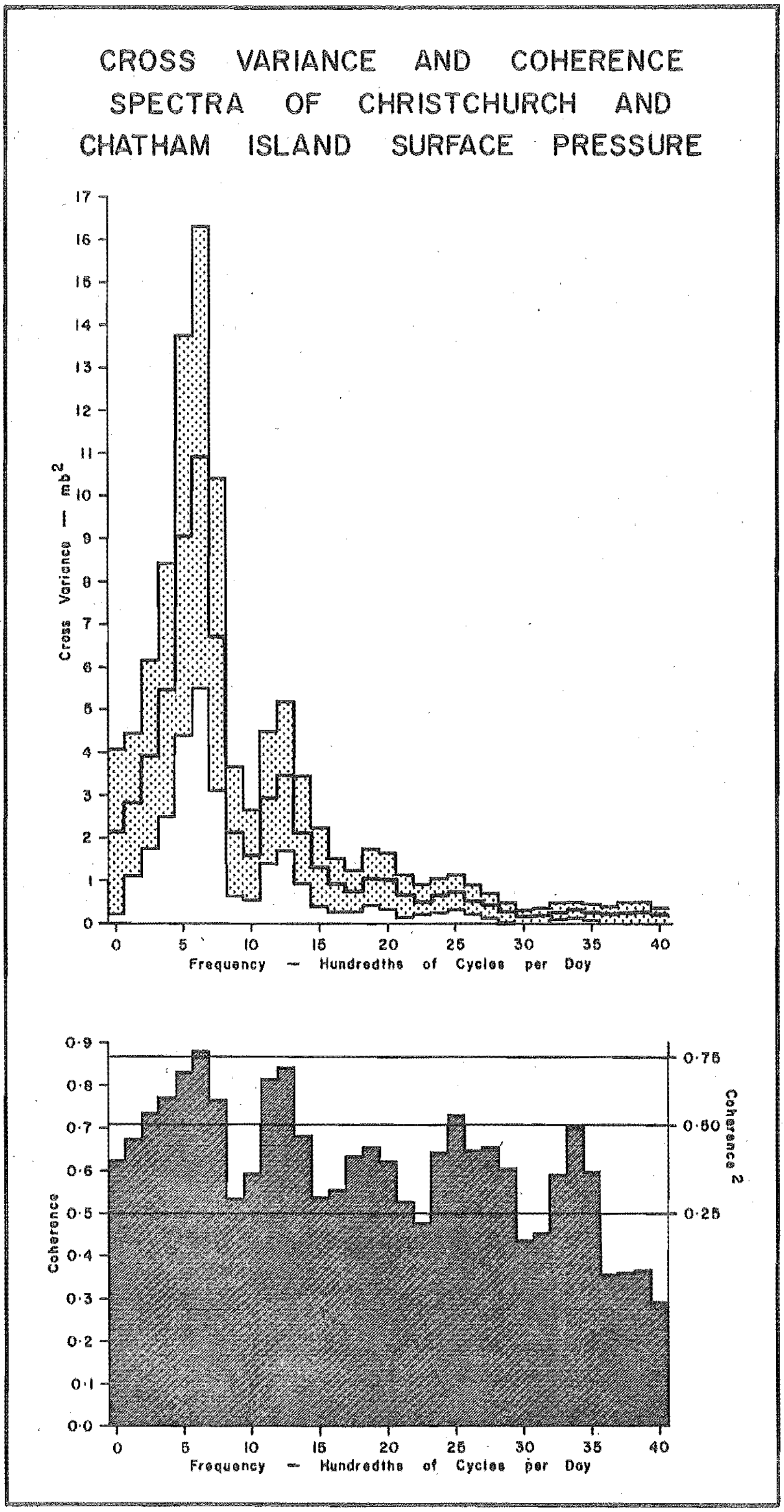
From the study of cross spectra of single elements at different places the shape, magnitude, homogeneity and rate of movement of systems may be assessed for a given frequency. Cross spectra of two different elements at the same place will reveal the internal structure of the systems. Such an approach requires the selection of particular frequencies for analysis. In the previous chapter it was seen that, whilst the spectra of individual elements were continuous, some frequencies appeared to be more important than others. A preliminary study of the cross spectra revealed that similar frequencies produced high correlation or coherence and large cross variance. For the present analysis 0.1375 cycles per day (7.27 days) and 0.0750 cycles per day (13.33 days) were selected for closer inspection.

The distinction between surface and upper air data will be maintained, since the former were smoothed and produced slightly different variance spectra from those of the upper air series. For the surface to upper air relationships the radiosonde surface data were used.

5.2 Surface Pressure

Of all the elements analysed in Chapter 4 it was seen that by far the most consistent was surface pressure. As might be expected, this carries over into the cross spectra between stations. Pressure, compared with for example, temperature, is little influenced by surface features so that differences between stations are due to changes within the advecting pressure systems themselves. As long as these changes are slow relative to the speed of translation there should be a high correlation between stations. That this is so is a well known fact, and is one of the reasons why pressure (or height) is the basic parameter used in forecasting.

Fig. 5.2.1 shows these relationships for two stations at similar latitudes for all the frequencies calculated. Throughout most of the frequencies over 25% of the individual variances appear in the cross variances. In several important frequencies this increases to over 50%. Frequencies of 0.0625 cycles



CROSS VARIANCE AND COHERENCE SPECTRA OF CHRISTCHURCH AND CHATHAM ISLAND SURFACE PRESSURE
Cross Variance — mb²
Frequency — Hundredths of Cycles per Day
Coherence
Coherence 2
0·75
0·50
0·25

Fig. 5.2.1

per day and 0.1250 cycles per day, which are slightly lower than those selected for specific attention, remain dominant as in the variance spectra. The 90% confidence bands, given for the cross variance, show that the peak at 0.0625 cycles per day is statistically significant.

The cross variances, coherences and lags in days for surface pressure have been plotted in Fig. 5.2.2 for a selection of stations at 0.0750 cycles per day. They show that south of about 25 S the pressure systems producing this frequency in the time data are fairly consistent. For several pairs of stations the square of the coherence is greater than 0.50; that is, the cross variance accounts for more than 50% of the individual variances. Also, despite the fact that this is not the most important frequency, the calculated lag-times fit very well together around triangles made up of three stations. Correlation between Nandi and the other stations is relatively low, but still the lag-times are consistent. However, the cross variances are low and not easily incorporated into the mid-latitude relationships.

From the lag-times the relative positions of some definable point in the pressure systems, such as a ridge line, may be plotted for successive days. From this analysis it appears that the pressure systems are on the average tilted from southwest to northeast, that they progress from west to east except in the north and that they tend to move more quickly in the east of New Zealand than they do in the west. The tilt is not in the expected direction but a discussion of this point will

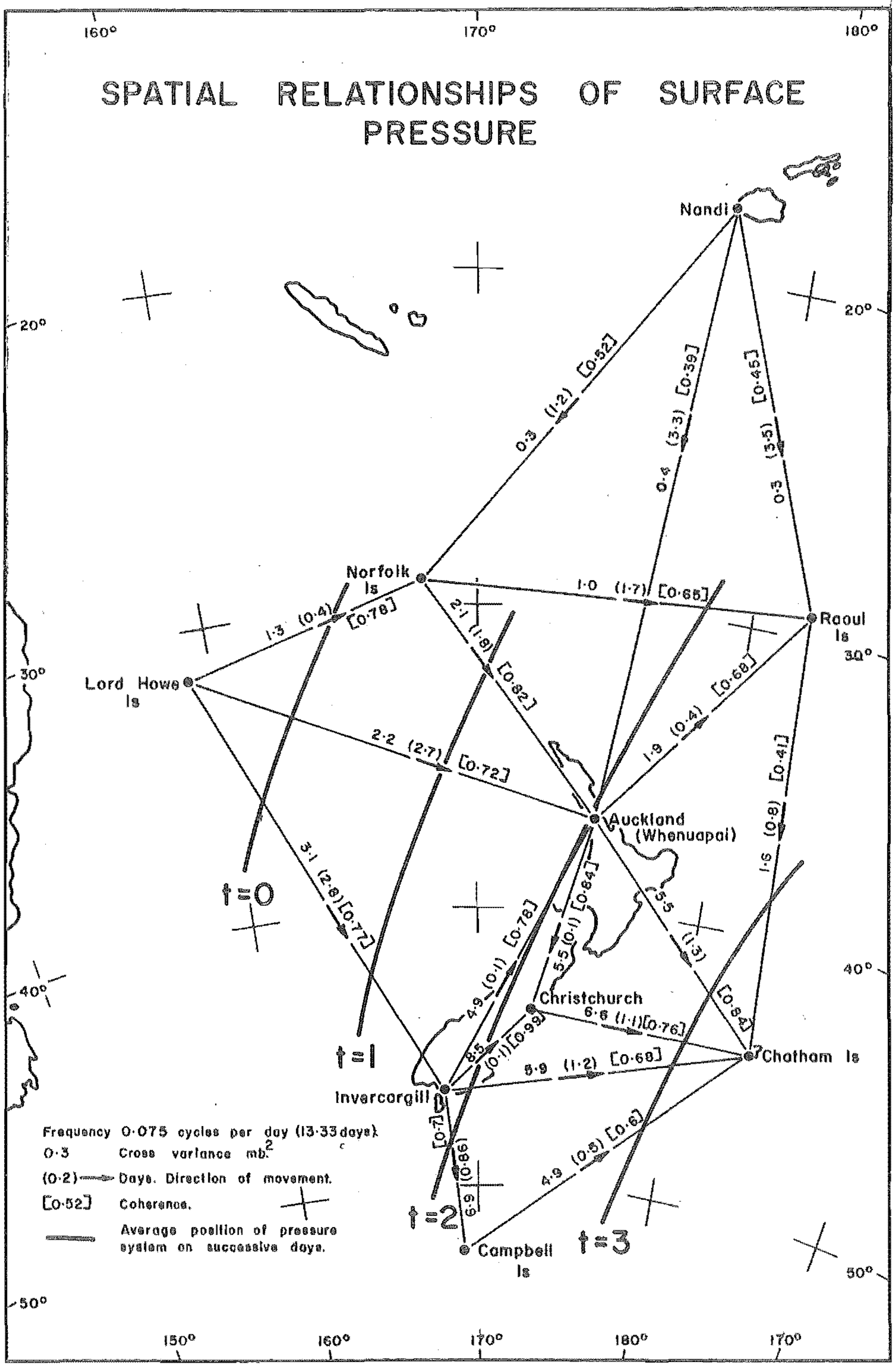

SPATIAL RELATIONSHIPS OF SURFACE PRESSURE
160°
170°
180°
20°
30°
40°
50°
150°
Nandi
Norfolk Is
Raoul Is
Lord Howe Is
Auckland (Whenuapai)
Christchurch
Chatham Is
Invercargill
Campbell Is
t=0
t=1
t=2
t=3
0·3 (1·2) [0·52]
0·4 (3·3) [0·39]
0·3 (3·5) [0·45]
1·3 (0·4) [0·78]
1·0 (1·7) [0·65]
2·1 (1·8) [0·82]
2·2 (2·7) [0·72]
1·9 (0·4) [0·68]
1·5 (0·8) [0·41]
3·1 (2·8) [0·77]
4·9 (0·1) [0·78]
5·5 (0·1) [0·84]
5·5 (1·3) [0·84]
6·6 (1·1) [0·76]
8·5 (0·1) [0·99]
5·9 (1·2) [0·68]
4·9 (0·5) [0·6]
6·9 (0·86) [0·7]
Frequency 0·075 cycles per day (13·33 days).
0·3 Cross variance mb.^{2}
(0·2) Days. Direction of movement.
[0·52] Coherence.
Average position of pressure system on successive days.

Fig. 5.2.2

be deferred until the next chapter. If the speed of movement is calculated from the Lord Howe - Whenuapai pair of stations a value of 5.7° of longitude per day is obtained. For a frequency of 0.0750 cycles per day this leads to a system which has the zonal dimension of about 72° longitude or, in zonal analysis terms, this is wavenumber 5. However, any correlation drawn with the spatial field must remain tentative.

A similar analysis was performed on 0.1375 cycles per day as shown in Fig. 5.2.3. The cross variances are considerably smaller here and the coherence is less significant. As a result, the lag-times are not so consistent and daily ridge positions are more difficult to draw. Nevertheless, it is apparent that there is still a southwest to northeast tilt although less distinct, that there is an eastward progression and that movement is slower in the east. Calculation from the same pair of stations shows that the speed of translation is about double that at 0.0750 cycles per day. This again suggests a wavelength of about 72° longitude.

Whilst a subjective analysis of daily weather maps shows that these calculated rates of movement are of the right order of magnitude, a check on the wavelength is more difficult and will not be persued further. In the Northern Hemisphere Eliasen (1958) has shown that wavenumber 5 was significant and did move towards the east, but with speeds varying between 0 and 9° longitude per day during a six week period in October and November

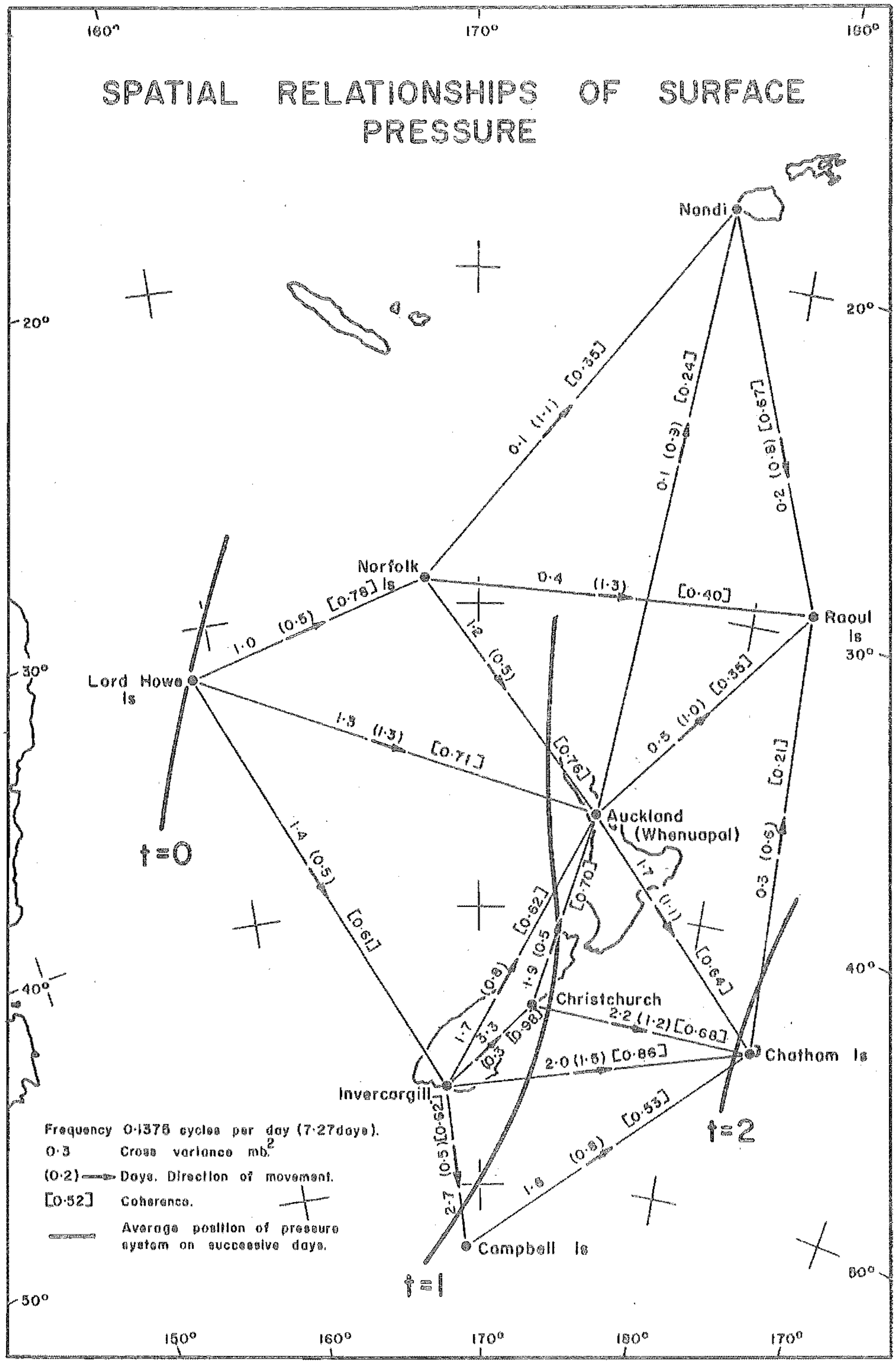

SPATIAL RELATIONSHIPS OF SURFACE PRESSURE
Nandi
Norfolk Is
Raoul Is
Lord Howe Is
Auckland (Whenuapai)
Christchurch
Chatham Is
Invercargill
Campbell Is
t=0
t=1
t=2
Frequency 0·1375 cycles per day (7·27days).
0·3 Cross variance mb.^{2}
(0·2) Days. Direction of movement.
[0·52] Coherence.
Average position of pressure system on successive days.

Fig. 5.2.3

1950. Similarly he found that wavenumbers 6 and 7 were present and moved with speeds between 11.7 and 16.7° longitude per day during the same months. It is feasible therefore for a wave of fixed length to produce peaks at more than one frequency in the time domain. It should be remembered also that the present calculations incorporate both winter and summer systems and speeds of translation, which have been shown to be different (Winston 1960, Harper 1961).

5.3 Surface Temperature, Dew Point and Wind

Spatial relationships of the other surface variables are not as clear as those of pressure. Local variations in the earth's surface now become important and the coherence between stations is generally lower. Consequently the lag calculations are less reliable and they tend to vary from the pressure pattern. The differences between the variables are shown in Table 5.3.1 for two pairs of stations. It is not clear whether the differing resultant speeds are real or whether they are due to sampling. The 90% confidence bands listed in Table 5.3.1 suggest that the lags might be the same, and that they all belong to one atmospheric system with the various elements remaining at the same phase with one another as the system moves across the area. If this is the case the maps presented in Figs. 5.2.2 and 5.2.3 will apply to surface temperature, dew point and wind.

5.4 Point Relationships between Surface Variables

Coherence and phase for two stations and the two frequencies are listed in Table 5.4.1. The phase between series reveals relatively high consistency despite the low coherence in a number of cases. They show that on the average at these frequencies north to south wind is correlated with high temperatures and south to north with low temperatures. Since dew point temperature and dry bulb temperature vary together, north to south wind is associated with high moisture content. Pressure and the V component of wind are about a quarter of a cycle out of phase which means that an east to west pressure gradient is associated with a south wind. Whilst this might have been anticipated in the upper air from the geostrophic wind relationship this need not necessarily follow at the ground surface where friction is a maximum.

The zonal wind has a similar relationship to pressure. This means that there is a fairly close correlation between the zonal and meridional components. It follows (see section 5.5) that the streamlines show an asymmetry with a tilt from southwest to northeast. There is an exception at Invercargill at 0.1375 cycles per day where the tilt is in the opposite direction.

5.5 Upper Wind Relationships

From the two wind components at any level a flow pattern may be evolved for a given frequency. For the individual elements the average amplitude for the

TABLE 5.3.1

COHERENCE AND LAG BETWEEN STATIONS
AT 0.0750 CYCLES PER DAY

Lord Howe – Whenuapai

	Coherence	Lag, days Estimated	90% Confidence band	
Pressure	0.72	2.7	2.0	3.4
Temperature	0.21	2.3	-0.2	5.4
Dew Point Temperature	0.46	2.3	1.0	3.6
Wind, u component	0.61	2.2	1.3	3.0
Wind, v component	0.36	2.6	0.8	4.4

Christchurch – Chatham Is.

	Coherence	Lag, days Estimated	90% Confidence band	
Pressure	0.76	1.1	0.5	1.6
Temperature	0.47	0.7	0.3	2.0
Dew Point Temperature	0.63	1.4	0.6	2.9
Wind, u component	0.47	2.0	0.7	3.3
Wind, v component	0.36	2.0	0.2	3.7

TABLE 5.4.1

COHERENCE AND PHASE BETWEEN
SURFACE VARIABLES

Whenuapai

	0.0750 cycles per day				0.1325 cycles per day			
		Phase, radians				Phase, radians		
	Coh.	Est.	90% Confidence		Coh.	Est.	90% Confidence	
Temp. – Pressure	0.61	3.75	3.32	4.17	0.63	3.91	2.68	5.14
Dew Pt. – Temp.	0.94	0.09	-0.02	0.21	0.79	0.05	-0.20	0.30
Wind u – Pressure	0.69	2.23	1.89	2.58	0.36	2.00	1.15	2.99
Wind v – Temp.	0.61	3.94	3.52	4.37	0.59	3.68	3.23	4.13
Wind v – Wind u	0.57	5.47	5.00	5.93	0.40	5.71	4.95	0.19
Wind v – Pressure	0.60	4.96	4.52	5.39	0.74	0.98	0.69	1.27
Wind u – Temp.	0.58	1.34	0.88	1.80	0.37	5.22	4.40	6.05

Invercargill

	0.0750 cycles per day				0.1375 cycles per day			
		Phase, radians				Phase, radians		
	Coh.	Est.	90% Confidence		Coh.	Est.	90% Confidence	
Temp. – Pressure	0.26	3.91	2.68	5.14	0.63	3.74	3.33	4.14
Dew Pt. – Temp.	0.95	0.10	-0.01	0.21	0.94	6.26	6.14	6.37
Wind u – Pressure	0.55	1.65	1.13	2.18	0.63	1.19	0.79	1.58
Wind v – Temp.	0.19	3.70	2.02	5.39	0.50	4.50	3.93	5.06
Wind v – Wind u	0.30	1.23	0.19	2.26	0.50	2.77	-0.29	0.84

frequency band is provided by the variance from the relationship

$$A_{(r)} = \sqrt{2 S_{(r)}}. \qquad 5.5\ (1)$$

For the south to north component, $V'_{(r)}$, this amplitude represents for a zero mean ($\bar{V} = 0$) an average south to north, north to south oscillation of wind (Figs. 5.5.1 and 5.5.2). If this is superimposed upon a mean uniform west to east current, $\bar{u}$, a simple symmetrical wave motion is set up. Now if u contains a fluctuation of the same average frequency, $u'_{(r)}$, this wave motion becomes distorted and assumes a shape dependent upon the relative amplitudes, $A_{u(r)}$ and $A_{v(r)}$ and the phase difference, $\Phi_{vu(r)}$, between them. Where the phase is $\frac{\pi}{2}$ the wave is symmetrical in the x direction regardless of the values of the amplitudes, but where $V'_{(r)}$ maximum follows $u'_{(r)}$ maximum by $\frac{\pi}{2}$ the trough line is sharp relative to the ridge, and where $u'_{(r)}$ follows $V'_{(r)}$ by the same amount the ridge becomes sharp. Where the phase differs from $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ the wave becomes asymmetrical. Theoretical shapes for such waves of varying phase angle have been calculated for $u'_{(r)}$ and $V'_{(r)}$ of unit amplitude superimposed upon a zonal current of speed 2. Equations for u and V are then

$$u = 2 + \cos(\theta + \Phi), \text{ and}$$

$$V = \cos\theta. \qquad 5.5\ (2)$$

Resulting waves in the spatial domain, drawn on the assumption that they have a uniform speed of translation,

VERTICAL VARIATION OF THE AMPLITUDES OF ZONAL AND MERIDIONAL WIND COMPONENTS AT 0·1375 CYCLES PER DAY

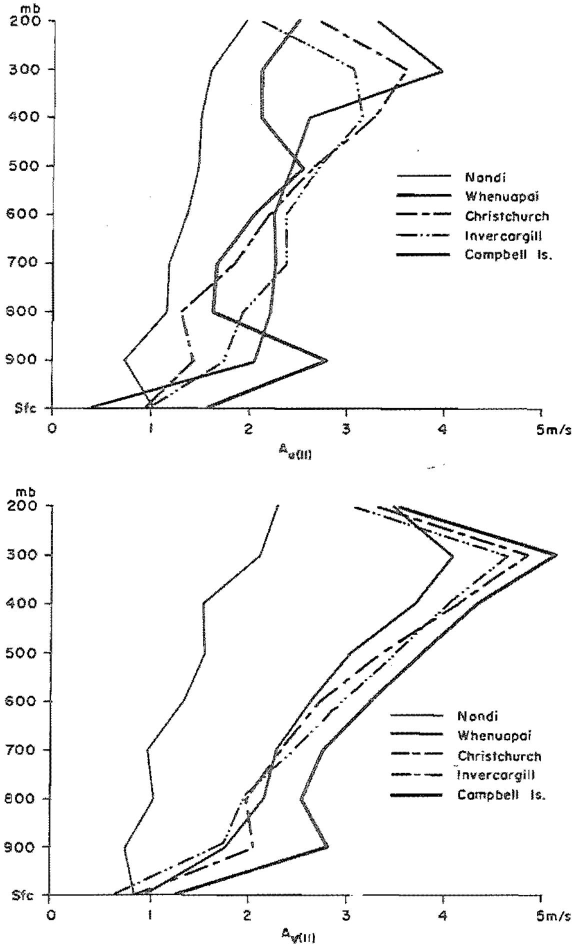


Fig. 5.5.1

VERTICAL VARIATION OF THE AMPLITUDES OF ZONAL AND MERIDIONAL WIND COMPONENTS AT 0·0750 CYCLES PER DAY

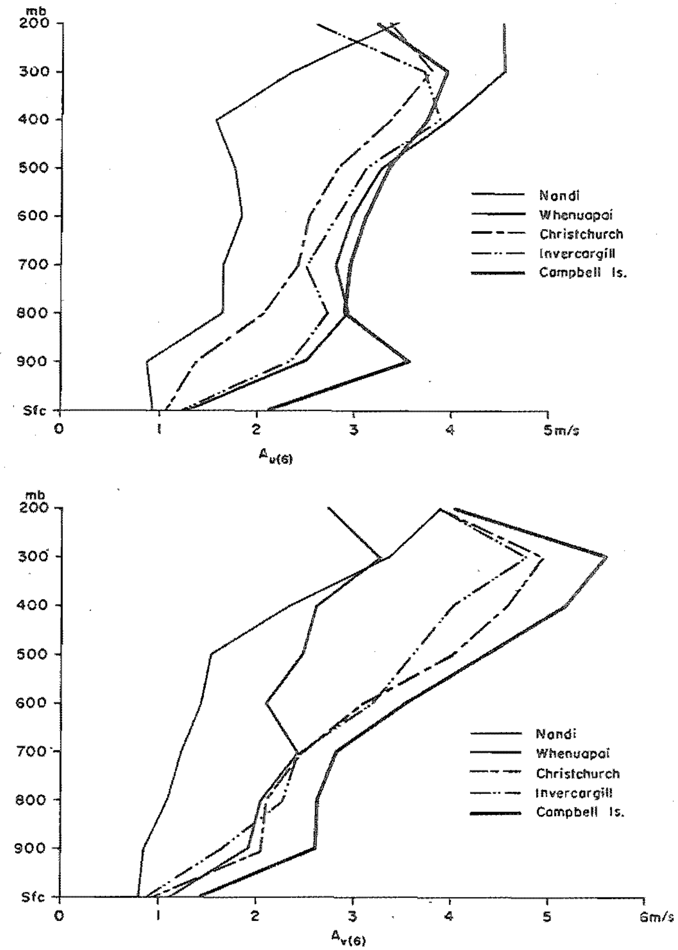


Fig. 5.5.2

are shown in Fig. 5.5.3. Whilst they represent the line of the wind they are not true streamlines since they are not formed from instantaneous observations.

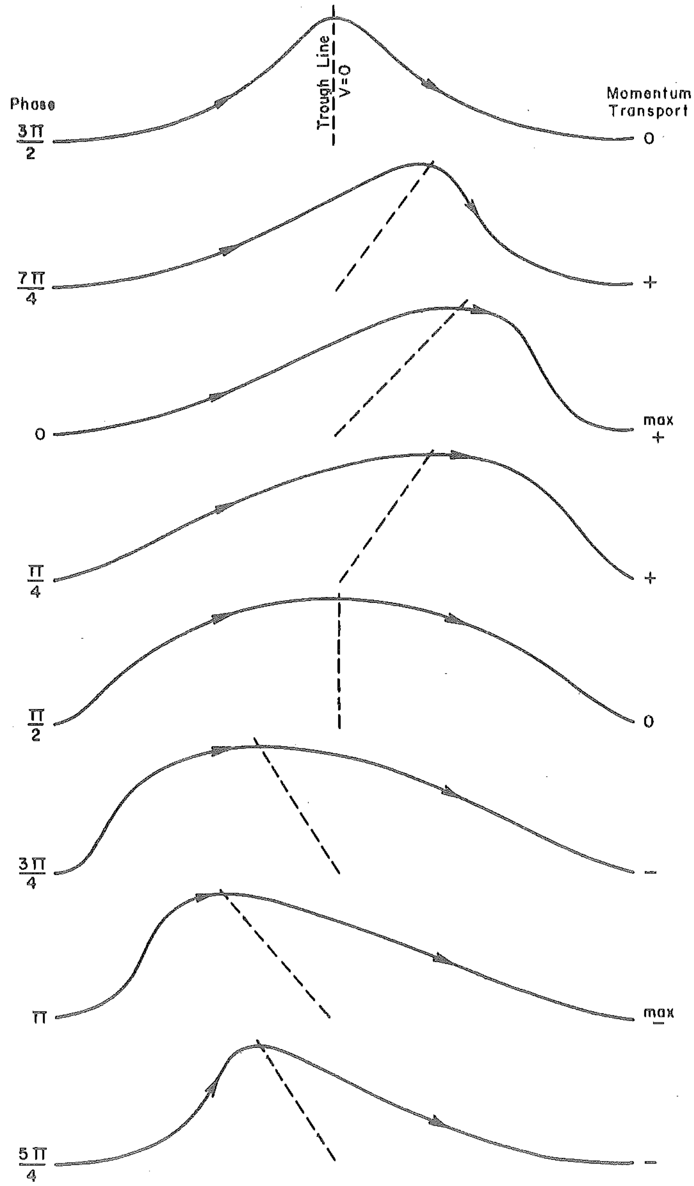
In the real atmosphere over the year's period of the present calculations the relationship between the two wind components is poor. It may be seen in Figs. 5.5.4 and 5.5.5 that generally this relationship accounts for less than 25% of the individual variances at all levels and stations. As a result the phase angles are very variable. At 0.1375 cycles per day except at Campbell Island the tilt of the waves is from northwest to southeast especially above 800 mb. At 0.0750 cycles per day only Nandi consistently records a northwest to southeast tilt, but the same alignment occurs at Whenuapai above 700 mb and at Christchurch and Invercargill above 300 mb.

In zonal Fourier analysis the heights of the pressure fields are usually analysed and the winds deduced from the geostrophic relationship. The tilt can then be calculated from the phase difference between waves at two latitudes. In the time domain, whilst the tilt may be inferred from this method, neither its magnitude nor the magnitude of the wind can be calculated unless the speed of translation is known accurately. In consequence no detailed analysis of the height field has been attempted.

5.6 Upper Air Temperature and Moisture

The amplitudes of temperature and moisture are

THEORETICAL SPATIAL ARRANGEMENT OF STREAMLINES
as calculated from phase differences between U and V



V = Cos θ U = 2 + Cos(θ − φ)

Fig. 5.5.3

VERTICAL VARIATION OF COHERENCE AND PHASE
BETWEEN V AND U
AT 0·1375 CYCLES PER DAY

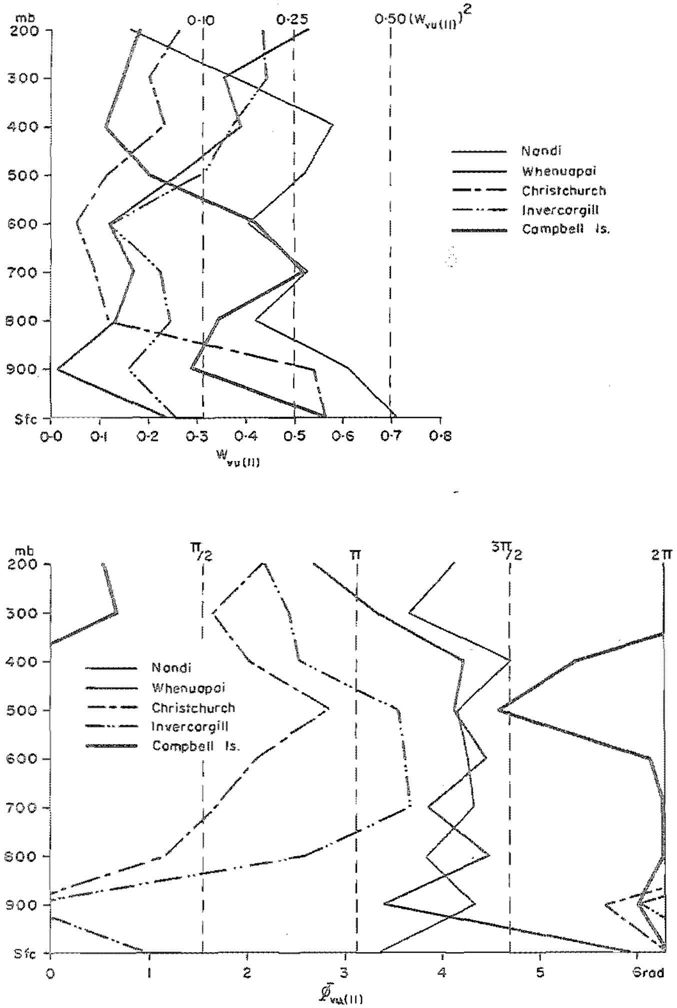


Fig. 5.5.4

VERTICAL VARIATION OF COHERENCE AND PHASE
BETWEEN V AND U
AT 0·0750 CYCLES PER DAY

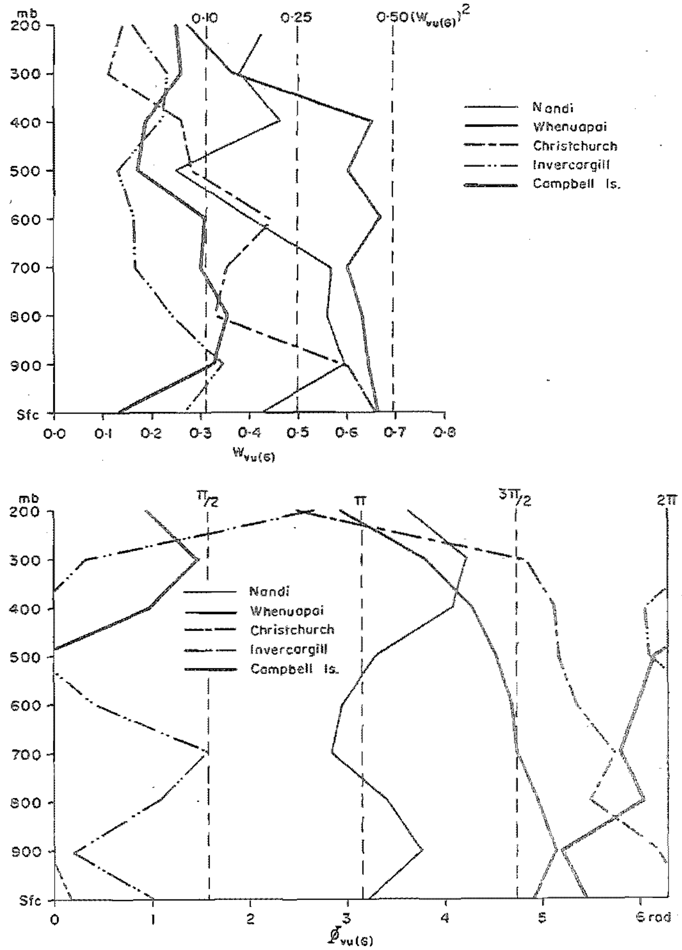


Fig. 5.5.5

plotted against height in Figs. 5.6.1 and 5.6.2 for 0.1375 and 0.0750 cycles per day. They follow the patterns of total variance already discussed in sections 4.8 and 4.9. Like the amplitude of meridional wind they may be considered as oscillations superimposed on the mean value. Since it has been shown (Figs. 3.7.1.1 and 3.7.2.1) that these averages increase towards the north, the deviations may be thought of as waves in the zonal pattern moving from west to east.

Interrelationships may again be gauged by the calculation of cross spectra between meridional wind and temperature, and meridional wind and moisture. The coherence between temperature and wind (Figs. 5.6.3 and 5.6.4) is higher than between the two wind components and lies often above 0.5 except at Nandi. There the amplitude of temperature is low and therefore not significant. This low correlation shows up in the phase angles which are erratic especially at 0.0750 cycles per day. The other stations show a uniform arrangement with height. At the surface, temperature is about π radians out of phase with the meridional wind which is in agreement with the more accurate surface synoptic data (section 5.4). There is agreement also at Invercargill where the phase angle is somewhat larger. This falls back to the same value as the others at 900 mb. Above that level there is a decrease in the phase to about 2 radians at 700 mb for the four southern stations. Between 700 and 400 mb at 0.1375 cycles per day and between 700 and 300 mb

VERTICAL VARIATION OF THE AMPLITUDES
OF TEMPERATURE AND MIXING RATIO
AT 0·1375 CYCLES PER DAY

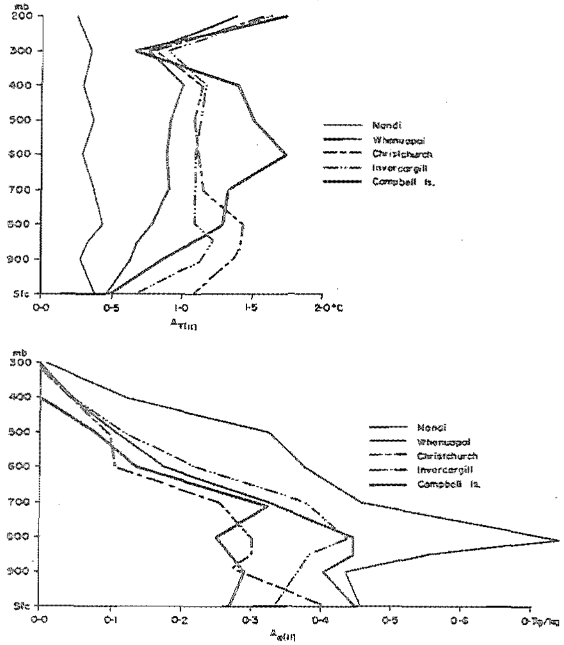


Fig. 5.6.1

VERTICAL VARIATION OF THE AMPLITUDES
OF TEMPERATURE AND MIXING RATIO
AT 0·0750 CYCLES PER DAY

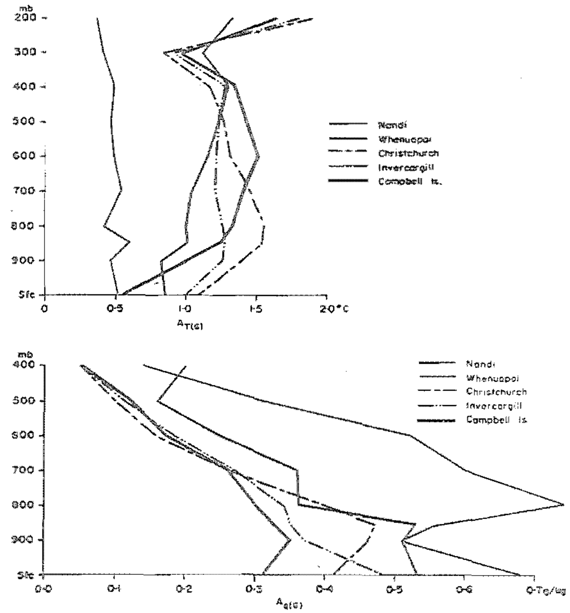


Fig. 5.6.2

VERTICAL VARIATION OF COHERENCE AND PHASE OF MERIDIONAL WIND AND TEMPERATURE AT 0·1375 CYCLES PER DAY

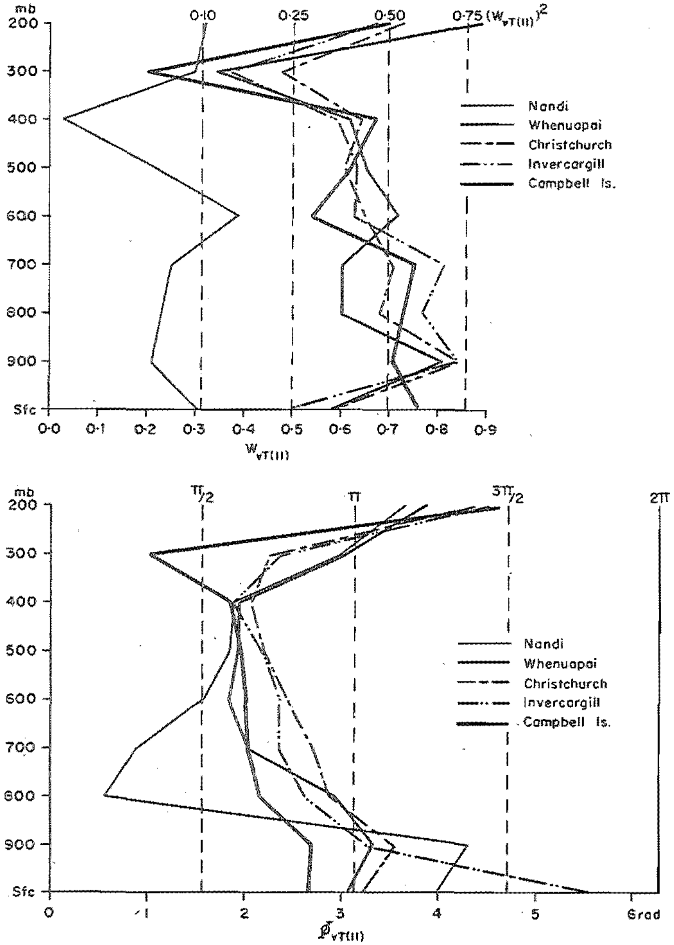


Fig. 5.6.3

VERTICAL VARIATION OF COHERENCE AND PHASE OF MERIDIONAL WIND AND TEMPERATURE AT 0·0750 CYCLES PER DAY

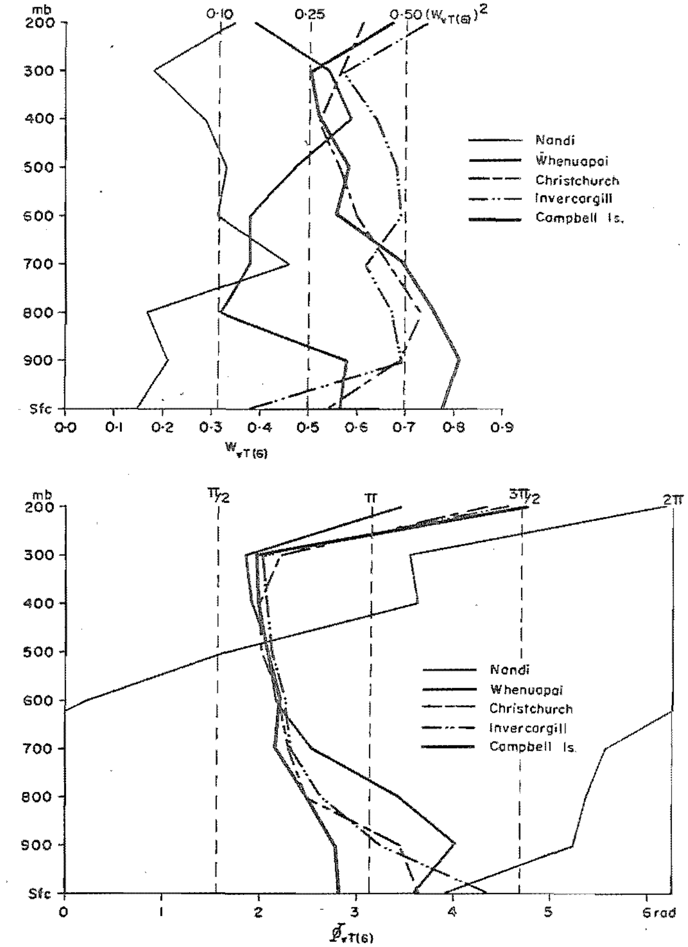


Fig. 5.6.4

at 0.0750 cycles per day there is little change in phase but above there is a sharp increase.

These relationships are shown in the spatial domain in Fig. 5.6.5. It is clear that so long as the phase remains between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ radians southerly winds will on an average be associated with low temperatures and northerly winds with high temperatures.

Coherence of wind with moisture is more variable than with temperature although it averages about 0.5. The amplitude of moisture is high at Nandi and the coherence is likewise more significant (Figs. 5.6.6. and 5.6.7). Again the phase angle is very variable with height and with latitude although the calculated values tend to cluster around π radians with a reduction of phase with height. It follows that temperature and moisture are almost in phase with one another. The spatial picture may be obtained from Fig. 5.6.5 with the replacement of temperature by moisture.

5.7 Solar Radiation and Precipitation

Whilst it might at first seem reasonable to correlate solar radiation and precipitation with other surface features there are obvious disadvantages. For example, the surface temperature and wind are very often dominated by local conditions which may mask the regional values. Furthermore, the dynamical factor which affects solar radiation and precipitation is vertical motion acting through cloud amount and water

THEORETICAL SPATIAL ARRANGEMENT OF SYMMETRICAL STREAMLINES AND TEMPERATURE (SOUTHERN HEMISPHERE)

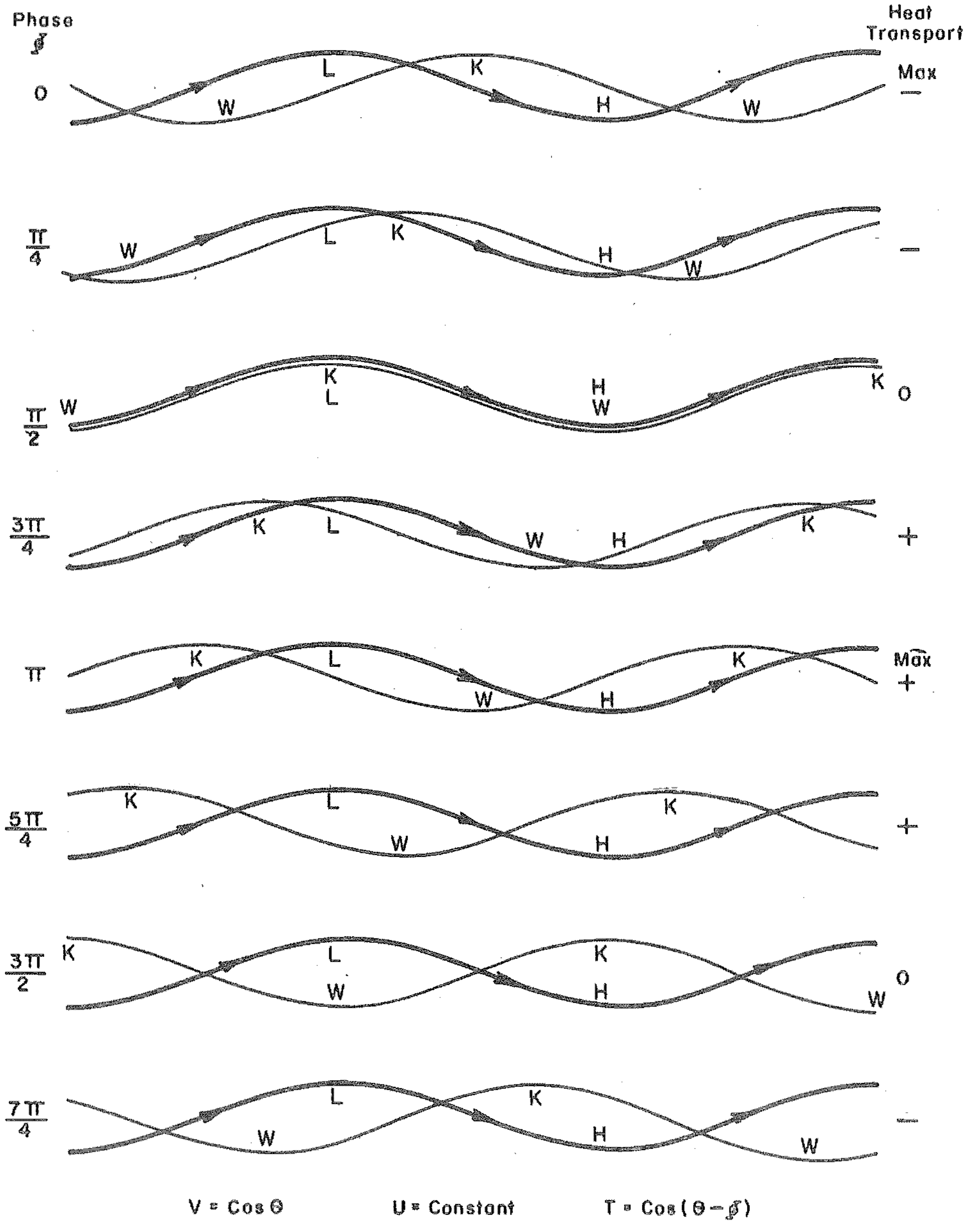


Fig. 5.6.5

VERTICAL VARIATION OF COHERENCE AND PHASE OF MERIDIONAL WIND AND MIXING RATIO AT 0·1375 CYCLES PER DAY

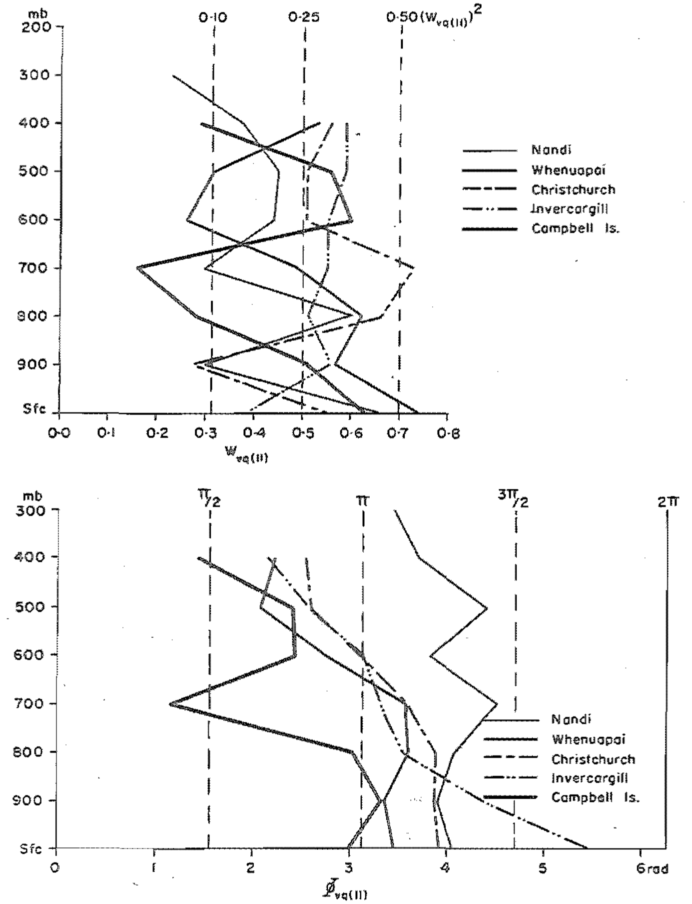


Fig. 5.6.6

VERTICAL VARIATION OF COHERENCE AND PHASE OF MERIDIONAL WIND AND MIXING RATIO AT 0·0750 CYCLES PER DAY

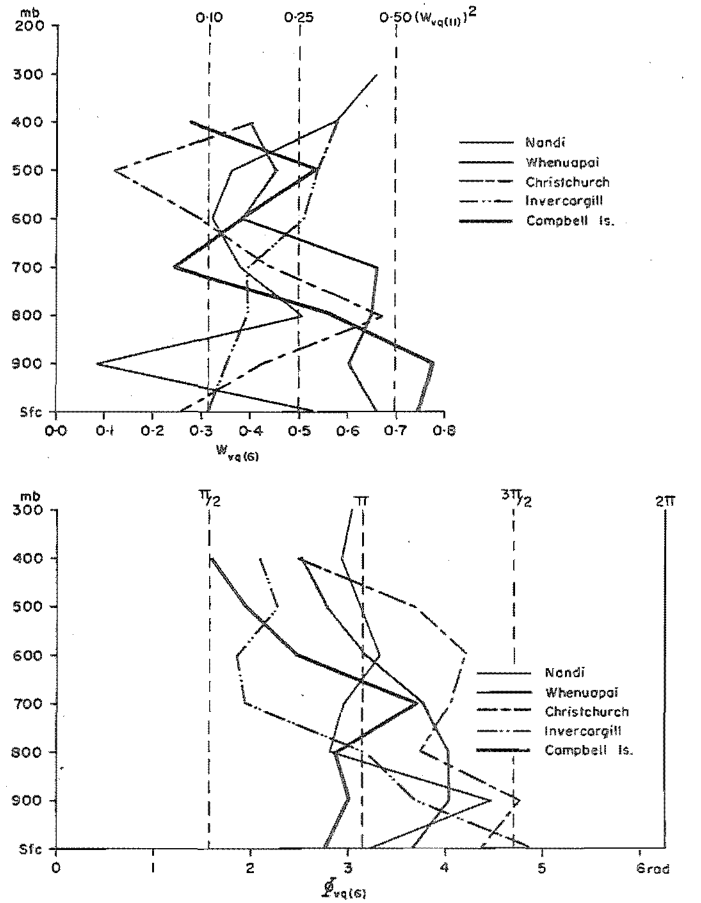


Fig. 5.6.7

vapour content. This vertical motion may occur at any level in the atmosphere but generally will be effective only below 500 mb. For the present study the 700 mb surface was selected for analysis although a lower level might have been more useful.

The phase relationships between meridional wind on the one hand and temperature, radiation and precipitation on the other are plotted in Fig. 5.7.1. The coherence is not high but it shows that high temperature and precipitation ~~is~~ are associated with north winds whilst high radiation is associated with south winds. This means that latent heat release associated with upward motion tends to increase the temperature of the warm air. The argument for radiation is less clear but it seems likely that this component tends to warm the cool air.

5.8 The Spatial Arrangement of the Model

It has been shown in the previous sections that there is internal consistency in the atmosphere between the variables of wind, temperature, moisture, solar radiation and precipitation. In other words, at the selected frequencies the atmosphere on the average is arranged in a particular, although not altogether unexpected, way. The western sides of ridges tend to be relatively warm and moist with upward motion and the eastern sides cool and dry. In the mid-troposphere the temperature and moisture are more in phase with the pressure field than at the surface or near the tropopause.

It should be noted that the time phase, ϕ_{xy} , gives the lag between the two variables x and y where x occurs before y. For two series passing through a point from west to east, x will be to the east of y.

From the cross spectra of variables at two levels the relative vertical positioning of these waves can be inferred. The use of temperature for such a calculation shows (Fig. 5.8.1) that there is a slight tilt towards the west from the surface to the mid-troposphere and then a reverse above. This amounts to about 1/6 of a wavelength at Whenuapai and somewhat less at Invercargill. Since wind also tilts to the west with height relative to temperature (Fig. 5.6.3) the associated pressure field is displaced nearly 1/3 of a wavelength to the west at 500 mb relative to the surface. This leads to an arrangement of temperature and pressure as shown in Fig. 5.8.2 which is in agreement with the thickness equation

$$\Delta z = \frac{R_d}{g} \bar{T}_v \ln \frac{p_1}{p_2}, \qquad 5.8\ (1)$$

where Δz is the thickness, R_d the gas constant for dry air, g the acceleration of gravity, p_1 and p_2 the pressures at the bottom and top of the layer and $\bar{T}_v$ the mean virtual temperature. Many text-books in meteorology, such as Haltiner et al. (1958) (page 210), explain this relationship. Added to Fig. 5.8.2 are the tentative vertical motions inferred from the precipitation data.

At 0.1375 cycles per day there is a tendency in

SPATIAL ARRANGEMENT OF TEMPERATURE, PRECIPITATION AND SOLAR RADIATION AT 700mb AT 0·1375 AND 0·0750 CYCLES PER DAY

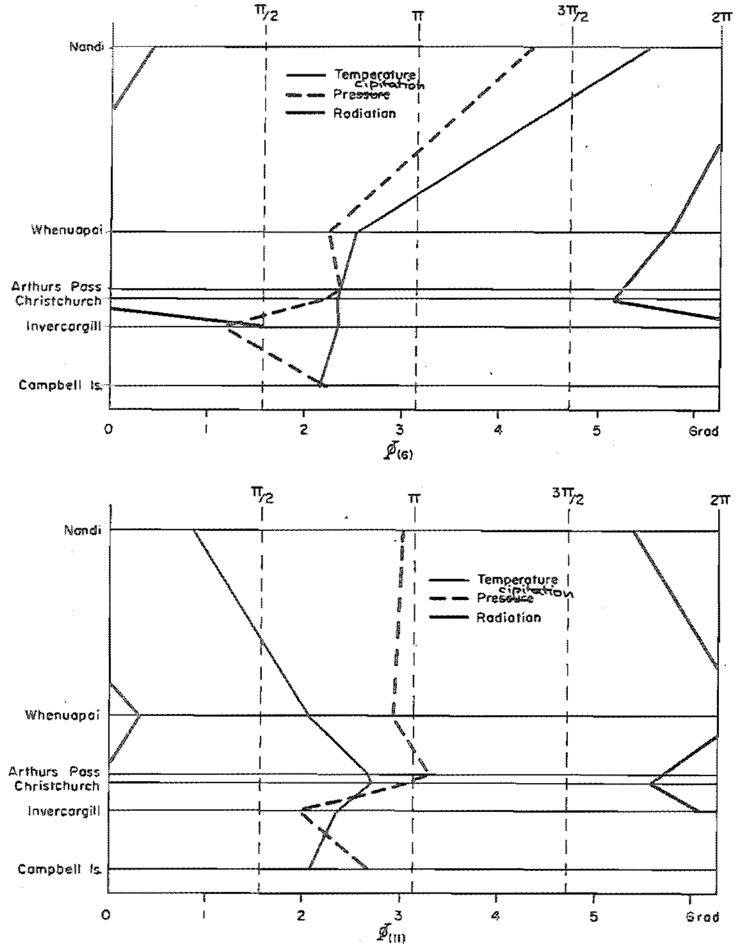


Fig. 5.7.1

VERTICAL TILT OF TEMPERATURE AT 0·1375 AND 0·0750 CYCLES PER DAY

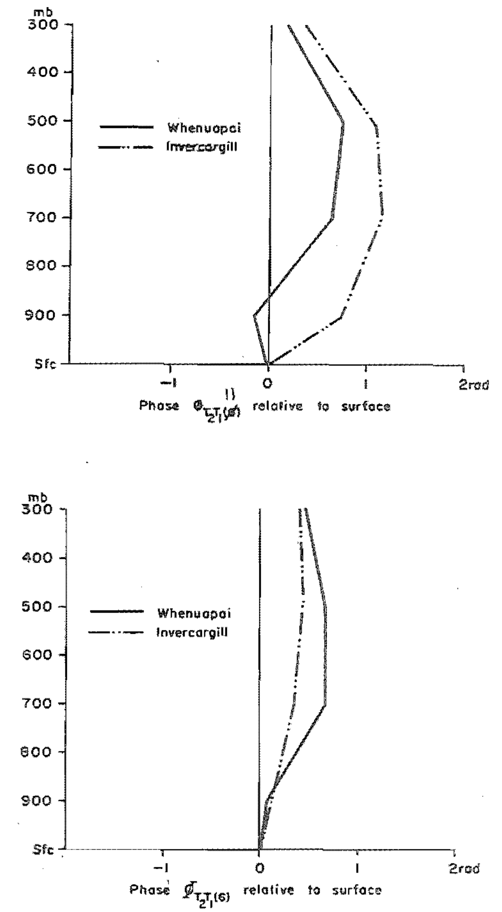


Fig. 5.8.1

the wind data (section 5.5) for a northwest to southeast tilt but the coherence is poor. Cross spectral analysis of temperature between stations reveals a tilt in the temperature field in the opposite direction at the four levels investigated (Fig. 5.8.3).

The same type of analysis at 0.0750 cycles per day gives similar results although the tilt with height and with latitude is less marked. The similarity between the two frequencies is not surprising if the fluctuations of the variables are due to the same wavelength travelling at different speeds as suggested in section 5.2. However, if there are waves of various lengths and stabilities the correspondence is remarkable. In one of the earliest attempts to assess the stability of waves in a baroclinic current Charney (1947) showed that the same temperature and wind arrangement as the one calculated here was unstable, and led to the development of waves 5 to 8. His neutral wave does not appear in the present analysis.

THE VERTICAL ARRANGEMENT OF TEMPERATURE AND PRESSURE AT 0·1375 CYCLES PER DAY

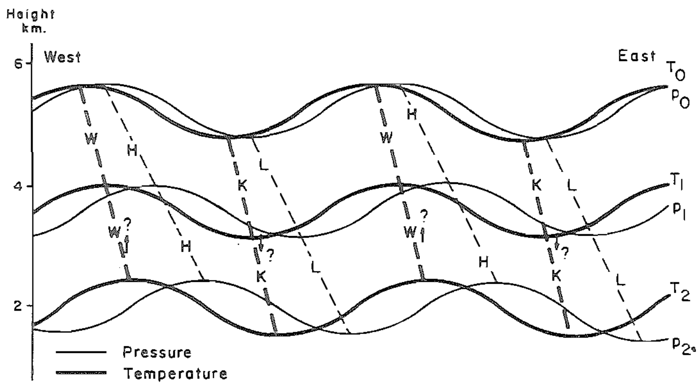


Fig. 5.8.2

HORIZONTAL TILT OF TEMPERATURE AT 0·1375 AND 0·0750 CYCLES PER DAY

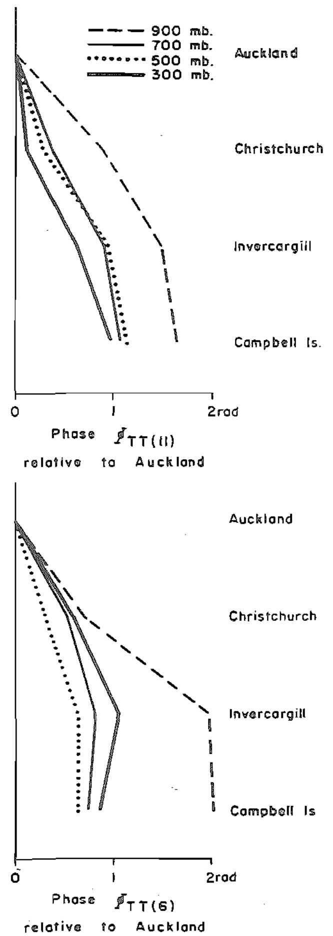


Fig. 5.8.3

CHAPTER 6

THE GENERAL CIRCULATION AND CLIMATE

6.1 The Balance Concept

Recently, with the increase in data, the basic physical principle of conservation has been used extensively in climatology at all scales. Since climatic entities such as the annual averages of temperature and zonal wind remain relatively constant the associated generative, dissipative and transfer processes must balance one another. For example, on a hemispheric scale, radiative heating in the tropical zone and cooling in higher latitudes must be balanced by a horizontal transfer of heat. As early as 1926 Jeffreys (1926) showed that a similar angular momentum transfer was necessary and how it could take place, but no one followed up this lead until the late 1940's when several groups made calculations of transfers (for example, Priestley 1951, Starr et al. 1954).

6.2. The Energy of the General Circulation

Basic to the general circulation is the balance of energy within the system. Of the total potential and internal energy in the atmosphere, only about 1/200th is available for conversion into kinetic energy. This has been termed available potential energy, AP_z, by Lorenz (1955). Since potential and internal energy remain in constant proportion to one another (for example, see Petterssen 1956, p. 309) they may be considered together as one term. The available potential energy may then be shown to have magnitude if the stratification is not both horizontal and statically stable. The generative processes for available potential energy are then essentially the unequal heating of the atmosphere which may at the same time increase or decrease the total potential energy.

On the other hand, as first shown by Margules (1903), "The kinetic energy of a mass of air is derived from its internal energy and from the work done by the force of gravity . . . . A system in which the masses are disturbed vertically from equilibrium can contain the necessary potential energy." In other words, large scale kinetic energy is derived from the available potential energy, although only about 1/10th of the kinetic energy possible is realized (Lorenz 1955).

Before the conversion process can be understood it has been found convenient to break the energies up into statistical zonal average and eddy components. Observation and theoretical calculations on simple barotropic and baroclinic models have shown that the atmosphere is inherently unstable and cannot exist for any length of time in a purely zonal arrangement. Consequently, eddy components are integral parts of the atmospheric circulation and such a statistical breakdown seems justified. Crude estimates of the four components for the Northern Hemisphere are given in rectangles in Fig. 6.2.1 from Boville (1961).

The zonal generation, GE_z, is proportional to the positive covariance of zonal temperature and heating: equatorial regions are warmed, polar regions are cooled. This serves to maintain the zonal available potential energy, AP_z. The conversion, CN_A, is carried out by the atmospheric eddies carrying relatively warm air towards the poles and cool air towards the equator. Statistically it is equivalent to a covariance between temperature and meridional wind. As pointed out in section 6.1 this transfer is also necessary to maintain average temperatures.

Most of the converted AP_z is lost through the negative eddy generating processes, GE_e; that is, the local warming of cold air and cooling of warm air in the same latitude (Brown 1964). Only about 1/10th of the energy derived through CN_A is converted into KE_e. This process, CN_e, is the sinking of cold air and the rising of warm air in the zonal plane. It requires the positive covariance of temperature and vertical motion. Unfortunately vertical motion is one of the most difficult variables to estimate in the atmosphere so this term is the most unreliable. Originally estimates came from theoretical models such as that of Fleagle (1957). More recently operational numerical prediction procedures have provided vertical motion data which have been used successfully by Wiin-Nielsen (1959, et al. 1963) and by Saltzman et al. (1960).

Both zonal and eddy kinetic energy will be lost

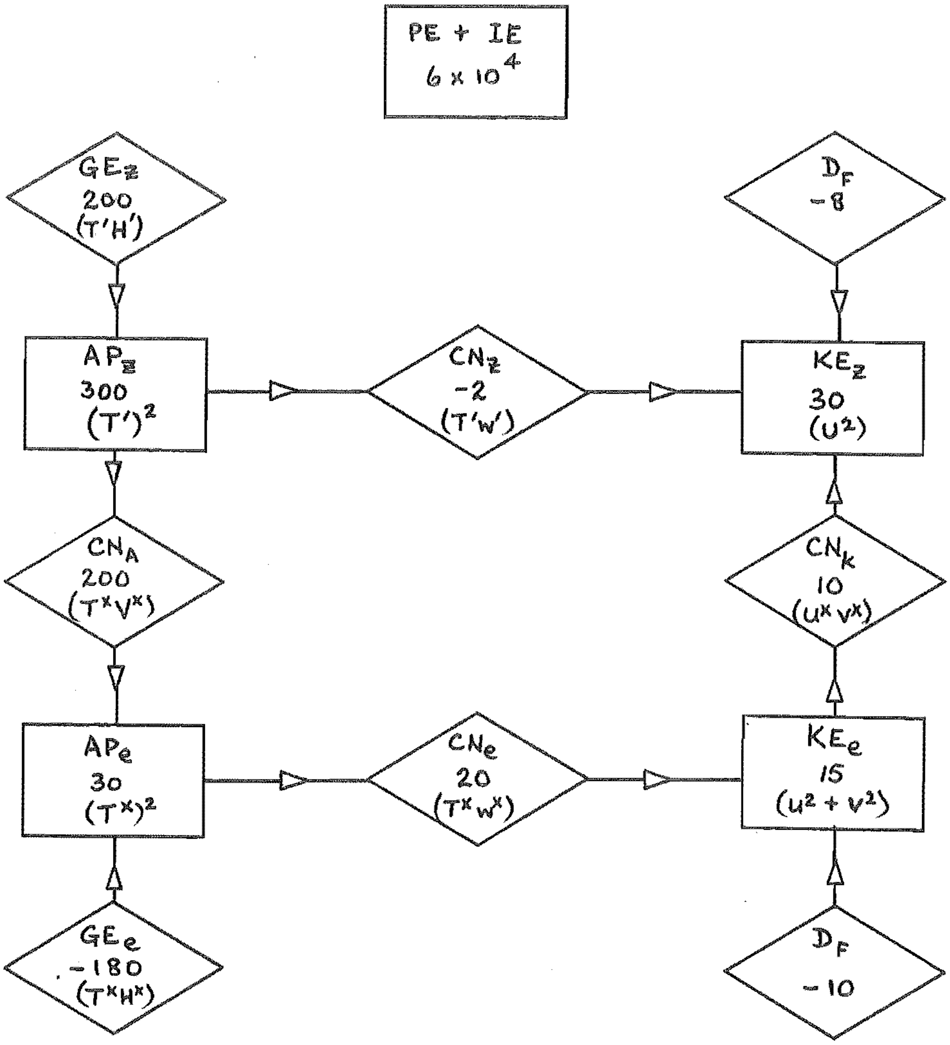


FIG. 6.2.1 (after Boville 1961, Fig. 32) SYMBOLIC REPRESENTATION OF THE ENERGY FLOWS IN THE GENERAL CIRCULATION - NORTHERN HEMISPHERE

Rectangles - energies in 10^{16} kj
Diamonds - energy conversion in 10^{10} kj sec^{-1}
′ deviations of latitude circle averages from the hemispheric averages
× deviations of the local value from the latitude circle average

by friction but whilst the eddy circulation is maintained by CN_e very little conversion takes place between the zonal available potential energy and zonal kinetic energy. Consequently there must be a conversion between the eddy and zonal components of kinetic energy. This is essentially accomplished by a transport of westerly angular momentum up the gradient of momentum.

From this brief summary it is clear that on a hemispheric basis the atmospheric circulation is driven by the latitudinal inequality of solar heating and that the eddies are an essential part of the energy conversion process. The individual transfer processes within this system have been studied since the late 1940's but only since 1955 have they been incorporated into the energy equations. In 1957 Saltzman (1957) derived the energy equations with respect to spatial scale using hemispheric Fourier analysis and since then effort has been directed towards the study of energy conversions in the domain of wavenumber (for example, Saltzman et al. 1964 and Wiin-Nielsen et al. 1964).

Deeper understanding of the dynamics of the general circulation has come therefore from the study of a spatial pattern and the time variation of that pattern. Regional climate on the other hand has here been approached from a study of the spatial variation of a time pattern. Already it has been shown that time-average disturbances at specific frequencies display features very similar to those in the spatial domain. In the next section it will be explained how the eddy transfer processes may be statistically calculated for one location and how they relate to the scale models of the previous chapter.

6.3 Method of Calculation of Atmospheric Transfers

By using a simple mathematical procedure, now often known as the Reynolds Resolution, several groups such as Starr et al. (1954) were able to break down the transfers into various statistical components which had physical reality.

The total south to north transfer of a commodity, s, through a latitudinal wall for a layer p_1 to p_2 may be written

$$\frac{1}{g}\int_{p_1}^{p_2}\int_0^x\int_0^t vs \; dt\,dx\,dp. \qquad 6.3\ (1)$$

The integration may be carried out in any order but results are more easily interpreted if the one above is used. By taking averages over time (indicated by a bar) and around a latitude circle (indicated by square brackets) this simplifies to

$$\frac{1}{g}\int_{p_2}^{p_1}[\overline{vs}]\; dp. \qquad 6.3\ (2)$$

Now if v and s are resolved into their Reynolds components

(with respect to time) $\quad v = \bar{v} + v'$

(with respect to W-E distance) $\quad \bar{v} = [\bar{v}] + \bar{v}^*$

(with respect to pressure) $\quad \underline{\underline{[\bar{v}]}} = \underline{\underline{\{[\bar{v}]\} + [\bar{v}]''}}$

Add, and $\quad v = v' + \bar{v}^* + [\bar{v}]'' + \{[\bar{v}]\}.$

$$6.3\ (3)$$

But if the time average is taken over a sufficiently long period $\{[\bar{v}]\} = 0$ from continuity considerations since average pressure at the surface does not change. In practice this term is seldom calculated to be zero and adjustments are usually necessary.

Similarly if S is broken into its time and longitudinal components,

$$s = s' + \bar{s}^* + [\bar{s}]. \tag{6.3 (4)}$$

The product of 6.3 (2) and 6.3 (4) averaged over time and longitude converts 6.3 (2) to

$$\frac{1}{g}\int_{p_1}^{p_2} [\overline{vs}]\,dp = \frac{1}{g}\int_{p_1}^{p_2} \left(\overline{v's'} \right) + [\bar{v}^*\bar{s}^*] + [\bar{v}]''[\bar{s}] \Big) dp. \tag{6.3 (5)}$$

The first term, $\overline{v's'}$, is the component of transfer due to the transient horizontal eddies, the second to the stationary waves and the third to the mean meridional circulations.

Calculations of the Northern Hemisphere transfers by various investigators show that the horizontal eddies are the most important, especially in mid-latitudes. Mean meridional circulations seem to dominate in the tropics. In the Southern Hemisphere lack of data makes calculations less reliable. The most recent results are by Obasi (1963) with I.G.Y. data which show that, in the case of momentum flux, the transient eddy component is one order of magnitude larger than the stationary waves and mean meridional flow both in the tropics and to the south. Whereas many studies of this type have been made into the average and seasonal transfers around a whole latitude circle necessary to maintain the general circulation, less is known of the local spatial and time variations. Studies by Starr et al. (1957) and Haines et al. (1963) suggest that in the Northern Hemisphere there are preferred longitudes and seasons of transport.

Spatial scale analysis of the eddies by way of Fourier analysis has been investigated by a number of authors and estimates are still appearing in the literature. Time scale analysis at single stations by Estoque (1955) and Chiu (1960) suggest that both low and high frequency components are important.

In the next section an attempt is made to divide the transfers over New Zealand into their various frequency components although, for one station alone, it is impossible to resolve the fluxes completely and equation 6.3 (5) reduces to

$$\frac{1}{g}\int_{p_1}^{p_2} (\overline{vs})\,dp = \frac{1}{g}\int_{p_1}^{p_2} \left(\overline{v's'} + \bar{v}\bar{s} \right) dp . \qquad 6.3\ (6)$$

The first term on the right, the instantaneous covariance of v and s, now represents the local transfer due to the transient horizontal eddies. This should remain close to the hemispheric average unless there are definitely preferred zones of transient eddy flux. The transfer may be thought of as being produced by the sum of the various scale model disturbances of section 5.8. In other words the time scale breakdown of $\overline{v's'}$ is given by the cospectrum.

The second term in 6.3 (6) is a combination of the local stationary and local meridional components and may now assume relatively large magnitudes. Most of the $\bar{s}$ carried north by $+\bar{v}$ in one area will be carried south again by $-\bar{v}$ further downstram.

6.4 Transfers 1962 - 1963

6.4.1 Relative Angular Momentum

The values of these calculations for the transient eddy flux are well below expected magnitudes and for Christchurch and Invercargill, at the lower levels, are in the "wrong" direction (Table 6.4.1.1, Fig. 6.4.1.1). That is, the transport does not offset the loss of westerly momentum by the surface stresses in mid-latitudes. Whilst the latitude averages for Campbell Island might be positive they are too large. Comparison with Priestley's (1951) figures for Auckland reveals good agreement up to 500 mb but thereafter a 3 to 1 discrepancy. For Obasi's (1963) latitudinal estimates the difference is in the ratio of 2 to 1.

This means, as shown in section 5.5, that on the average during the year August 1962 - August 1963 over New Zealand the waves were tilted from southwest to northeast at low levels and from northwest to southeast at high levels. According to Obasi the maximum transport occurs in the region of Auckland at about 250 mb.

Since there are two maxima of westerly momentum it is not clear whether the conversion CN_K is directed towards the zonal or eddy motion.

TABLE 6.4.1.1

RELATIVE ANGULAR MOMENTUM TRANSFER

$\times 10^{14}$ gm cm^2 sec^{-1} (100mb cm sec)$^{-1}$

	Transient Eddy $\overline{V'U'}$	Annual Eddy Component	Total Eddy	Mean Component
Nandi				
Surface	- 30.1	- 3.1	- 33.2	- 0.5
900 mb	- 15.3	- 1.2	- 16.5	+ 8.2
800	- 36.9	- 0.2	- 37.1	+ 1.8
700	- 34.2	+ 0.7	- 33.5	- 24.1
600	- 32.6	- 5.2	- 37.8	- 57.7
500	- 54.7	- 14.8	- 69.5	- 84.8
400	- 56.9	- 26.7	- 83.6	- 83.3
300	- 76.9	- 30.0	- 106.9	- 8.1
200	- 184.4	- 27.9	- 212.3	+ 88.9
Whenuapai				
Surface	+ 27.4	- 0.3	+ 27.1	- 0.3
900 mb	+ 57.3	- 4.1	+ 53.2	- 8.9
800	+ 14.8	+ 4.4	+ 19.2	- 2.6
700	- 5.1	- 3.9	- 9.0	+ 4.0
600	- 17.2	- 4.3	- 21.5	- 5.9
500	- 25.2	- 4.7	- 29.9	- 11.8
400	- 33.2	- 8.8	- 42.0	- 49.9
300	- 107.6	- 23.0	- 130.6	- 114.1
200	- 136.3	- 89.0	- 225.3	- 60.6
Christchurch				
Surface	+ 26.7	+ 1.0	+ 27.7	+ 0.0
900 mb	+ 58.8	+ 0.4	+ 59.2	- 3.9
800	+ 16.0	- 3.4	+ 12.6	- 10.8
700	+ 17.9	- 2.3	+ 15.6	+ 5.6
600	+ 39.9	+ 0.8	+ 40.7	+ 22.2
500	+ 32.9	+ 2.3	+ 35.2	+ 12.6
400	+ 23.0	+ 0.6	+ 23.6	- 29.5
300	+ 6.0	+ 7.3	+ 13.3	- 32.6
200	- 34.1	+ 5.6	- 28.5	- 23.7
Invercargill				
Surface	+ 9.0	+ 1.8	+ 10.8	- 7.2
900 mb	+ 31.0	- 0.1	+ 30.9	- 47.0
800	- 12.2	- 4.0	- 16.2	- 80.5
700	- 9.8	- 2.3	- 12.1	- 43.4
600	+ 13.0	- 5.2	+ 7.8	- 56.0
500	+ 21.9	- 7.8	+ 14.1	- 74.3
400	+ 35.3	- 5.9	+ 29.4	- 133.0
300	- 26.6	- 12.8	- 39.4	- 171.9
200	- 39.8	+ 3.8	- 36.0	- 105.5
Campbell Is.				
Surface	+ 25.0	+ 0.4	+ 25.4	- 14.7
900 mb	+ 20.8	- 0.9	+ 19.9	- 100.5
800	+ 64.9	- 1.3	+ 63.6	- 73.6
700	+ 60.0	- 2.9	+ 57.1	- 73.0
600	+ 83.3	- 1.9	+ 81.4	- 114.5
500	+ 87.4	- 2.1	+ 85.3	- 112.0
400	+ 142.0	+ 1.5	+ 143.5	- 156.5
300	+ 154.4	+ 5.2	+ 159.6	- 112.7
200	+ 94.3	- 0.7	+ 93.6	- 154.4

The annual components, as given by

$$A_v A_u \cos \Phi_{vu}, \qquad 6.4\ (1)$$

are negative, except for Christchurch and at high levels for the southern stations, but do not make up the apparent difference. A check on the zonal wind during the period shows no radical variation from normal (section 3.7.3) so the values of eddy flux in the longitude of New Zealand must be anomalies in the hemispheric pattern.

In magnitude the annual components are relatively large and would dominate if added to the spectral plots. The average cospectrum for each station is given in Figs. 6.4.1.2 to 6.4.1.6. The upper and lower limits are the extreme values recorded from the nine levels. It has already been pointed out that the coherence between the wind components is poor (section 5.5). Consequently the spectra are not very reliable. Large negative values occur at Nandi in the first seven frequency bands and negative peaks at Whenuapai are recorded at 0.0500 and 0.1250 cycles per day. Positive peaks may be observed at 0.0250 and 0.0625 cycles per day in the south.

The transport by the mean term is added for completeness in Table 6.4.1.1. This neglects the Ω term of absolute momentum transfer which has magnitude by virtue of a mean mass transport.

EDDY TRANSFER OF RELATIVE ANGULAR MOMENTUM
AUGUST 1962–AUGUST 1963

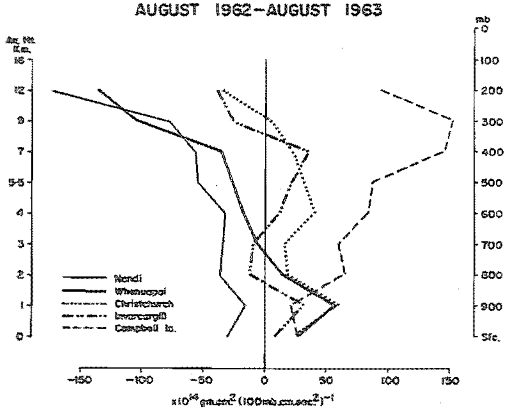


Fig. 6.4.1.1

RELATIVE ANGULAR MOMENTUM FLUX SPECTRUM
NANDI

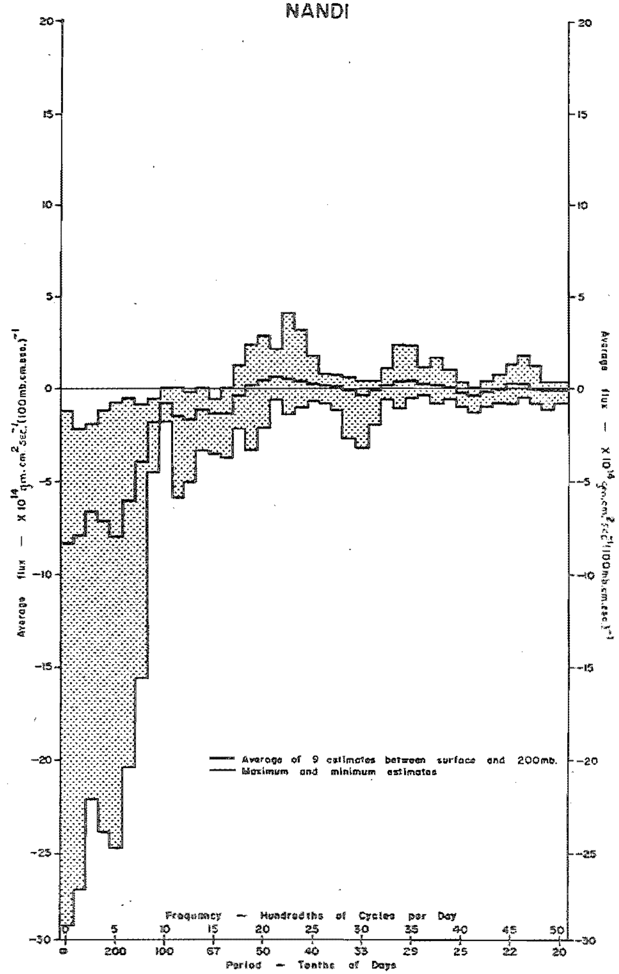


Fig. 6.4.1.2

RELATIVE ANGULAR MOMENTUM FLUX SPECTRUM WHENUAPAI

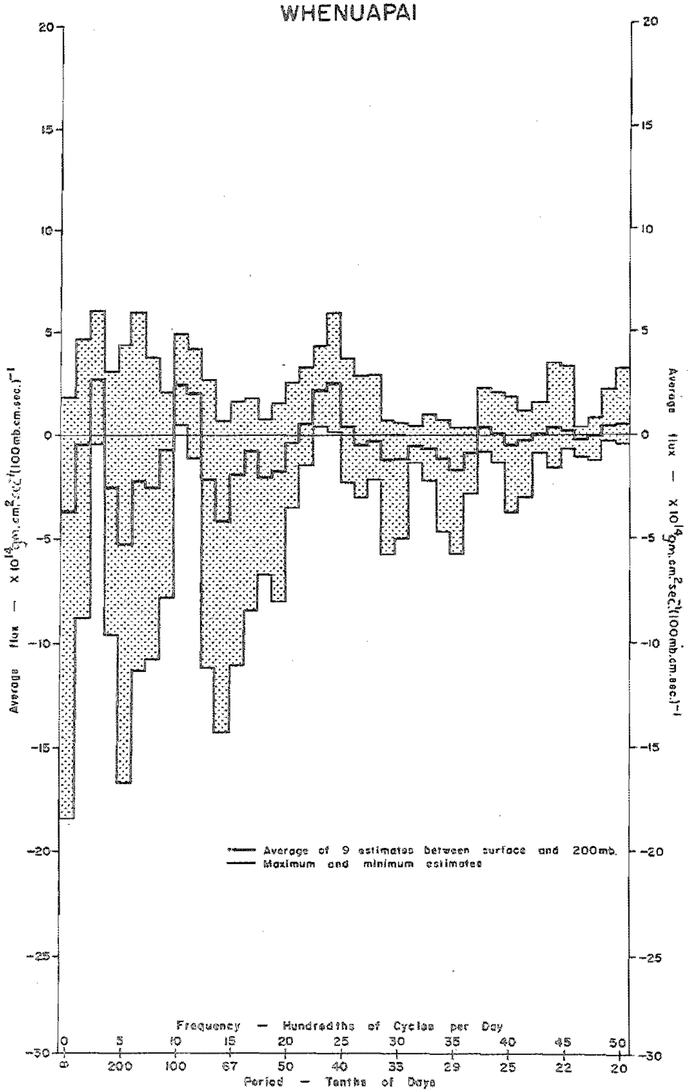


Fig. 6.4.1.3

RELATIVE ANGULAR MOMENTUM FLUX SPECTRUM CHRISTCHURCH

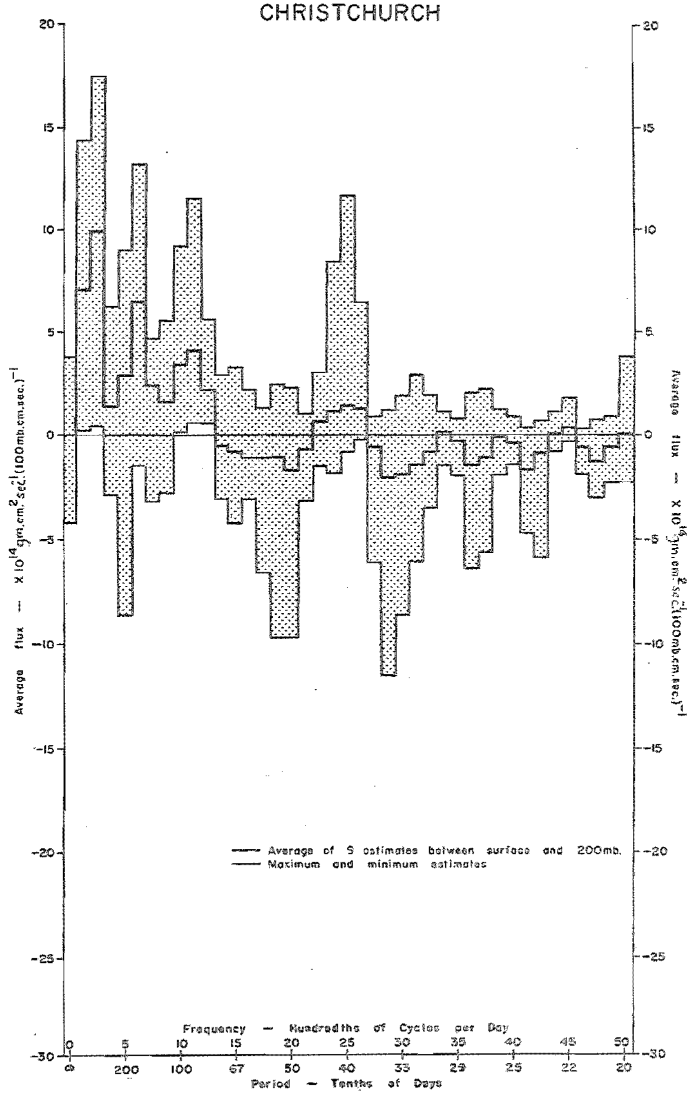


Fig. 6.4.1.4

RELATIVE ANGULAR MOMENTUM FLUX SPECTRUM
INVERCARGILL

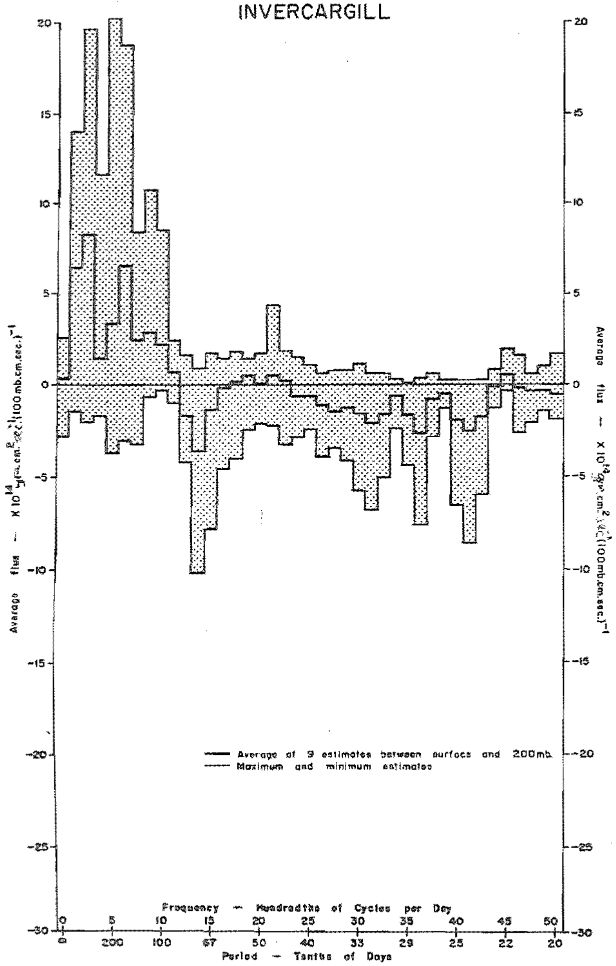


Fig. 6.4.1.5

RELATIVE ANGULAR MOMENTUM FLUX SPECTRUM
CAMPBELL ISLAND

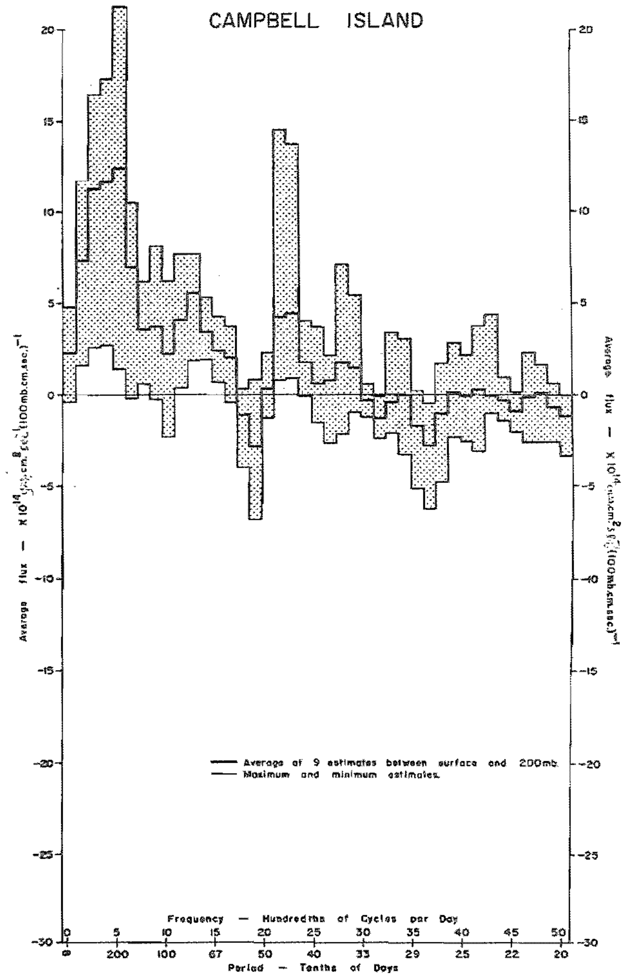


Fig. 6.4.1.6

6.4.2 Sensible Heat

The transient eddies transport sensible heat southwards at all four southern stations at all levels as might be expected (Table 6.4.2.1, Fig. 6.4.2.1). At Nandi the transfer is small and may be neglected. The maximum flux occurs at Christchurch and Campbell Island between 900 and 800 mb. At Christchurch this is associated with relatively high temperature variance whilst at Campbell Island with high wind variance. At all stations a definite minimum occurs between 400 and 500 mb where the phase angle between v and T becomes small.

The transfer by the annual oscillation is relatively large and makes a significant contribution to the southward flux at Christchurch, Invercargill and Campbell Island. At Whenuapai it is directed northwards owing to the phase relationship, and at Nandi this component is small because the annual variation of temperature is negligible.

The spectra of sensible heat transfer show a sharp cut-off for periods shorter than 5 days except at Campbell Island at high levels where the covariances become erratic. The negative peaks at most levels at about 0.0625, 0.1375 and 0.2250 cycles per day carry over into the average spectra of Figs. 6.4.2.2. to 6.4.2.6.

Relatively, the mean flux term is extremely large despite small average wind speeds. This is due to small fluctuations in absolute temperature in the atmosphere compared to the average.

TABLE 6.4.2.1

SENSIBLE HEAT TRANSFER

$\times 10^3$ cal (100 mb cm sec^{-1})

	Transient Eddy $\overline{V'T'}$	Annual Eddy Component	Total Eddy	Mean Component
Nandi				
Surface	- 1.8	- 1.0	- 2.8	- 104.0
900 mb	- 2.3	- 1.3	- 3.6	- 590.3
800	+ 0.3	- 0.2	+ 0.1	- 1134.2
700	+ 1.3	+ 0.0	+ 1.3	- 1225.1
600	+ 1.8	+ 1.7	+ 3.5	- 1278.1
500	- 1.5	+ 1.1	- 0.4	- 1266.0
400	- 1.1	+ 1.4	+ 0.3	- 791.0
300	- 4.2	+ 0.5	- 3.7	- 50.1
200	+ 3.3	+ 0.1	+ 3.4	+ 398.4
Whenuapai				
Surface	- 6.6	+ 0.1	- 6.5	- 60.3
900 mb	- 15.5	+ 2.6	- 12.9	- 419.9
800	- 16.0	+ 1.0	- 15.0	- 77.1
700	- 13.8	+ 5.5	- 8.3	+ 94.8
600	- 11.6	+ 5.3	- 6.3	- 109.8
500	- 16.5	+ 7.3	- 9.2	- 156.8
400	- 18.7	+ 8.8	- 9.9	- 476.6
300	- 21.8	+ 12.2	- 9.6	- 788.9
200	- 22.1	- 17.7	- 39.8	- 299.2
Christchurch				
Surface	- 14.0	- 2.7	- 16.7	+ 107.7
900 mb	- 37.2	- 5.1	- 42.3	- 375.3
800	- 35.0	- 4.2	- 39.2	- 325.9
700	- 33.5	- 4.0	- 37.5	+ 131.7
600	- 25.5	- 0.6	- 26.1	+ 398.9
500	- 19.8	- 1.9	- 21.7	+ 171.6
400	- 24.9	- 3.8	- 28.7	- 312.8
300	- 28.5	+ 7.8	- 20.7	- 264.8
200	- 35.5	- 6.9	- 42.4	+ 168.7
Invercargill				
Surface	- 7.9	+ 5.2	- 2.7	- 451.7
900 mb	- 29.9	- 2.4	- 32.3	- 1152.2
800	- 27.9	- 3.8	- 31.7	- 1346.7
700	- 21.4	- 2.5	- 23.9	- 653.7
600	- 22.9	- 4.3	- 27.2	- 717.5
500	- 19.0	- 7.4	- 26.4	- 854.4
400	- 17.4	- 5.4	- 22.8	- 1254.5
300	- 25.1	- 7.7	- 32.8	- 1381.6
200	- 35.8	- 1.3	- 37.1	- 751.5
Campbell Is.				
Surface	- 13.9	- 3.7	- 17.6	- 435.2
900 mb	- 39.6	- 5.0	- 44.6	- 1438.6
800	- 40.6	- 4.3	- 44.9	- 915.7
700	- 29.1	- 7.0	- 36.1	- 848.6
600	- 31.4	- 7.6	- 39.0	- 1153.4
500	- 15.7	- 7.8	- 23.5	- 985.6
400	- 6.9	- 5.0	- 11.9	- 1130.3
300	- 13.7	+ 2.4	- 11.3	- 686.5
200	- 25.3	- 7.2	- 32.5	- 918.1

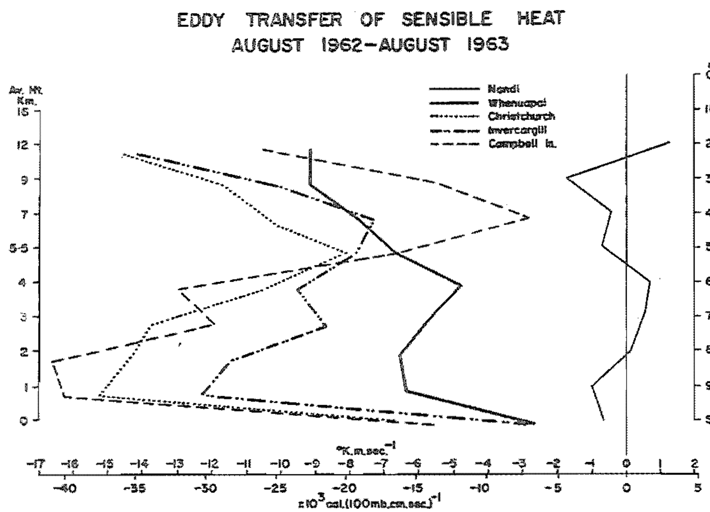


Fig. 6.4.2.1

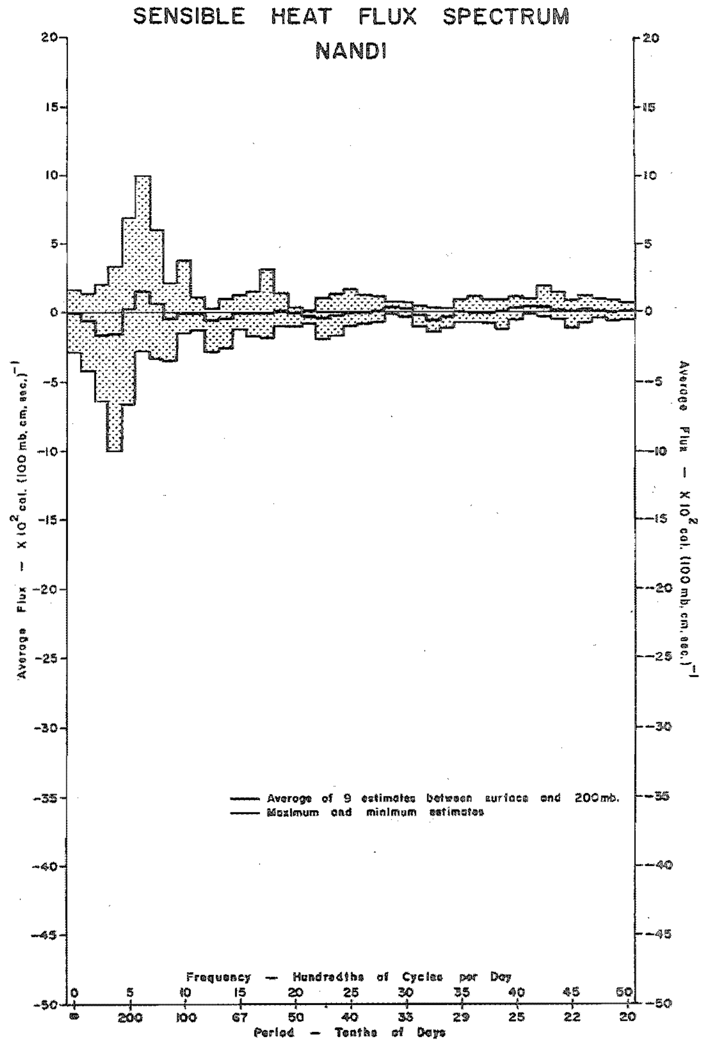


Fig. 6.4.2.2

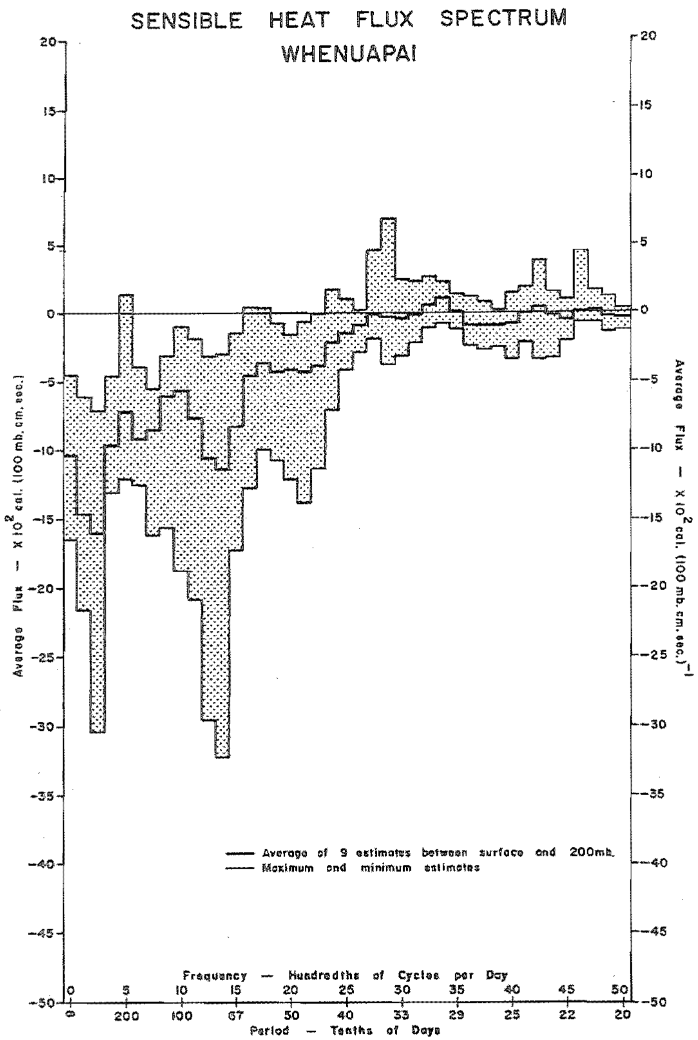


Fig. 6.4.2.3

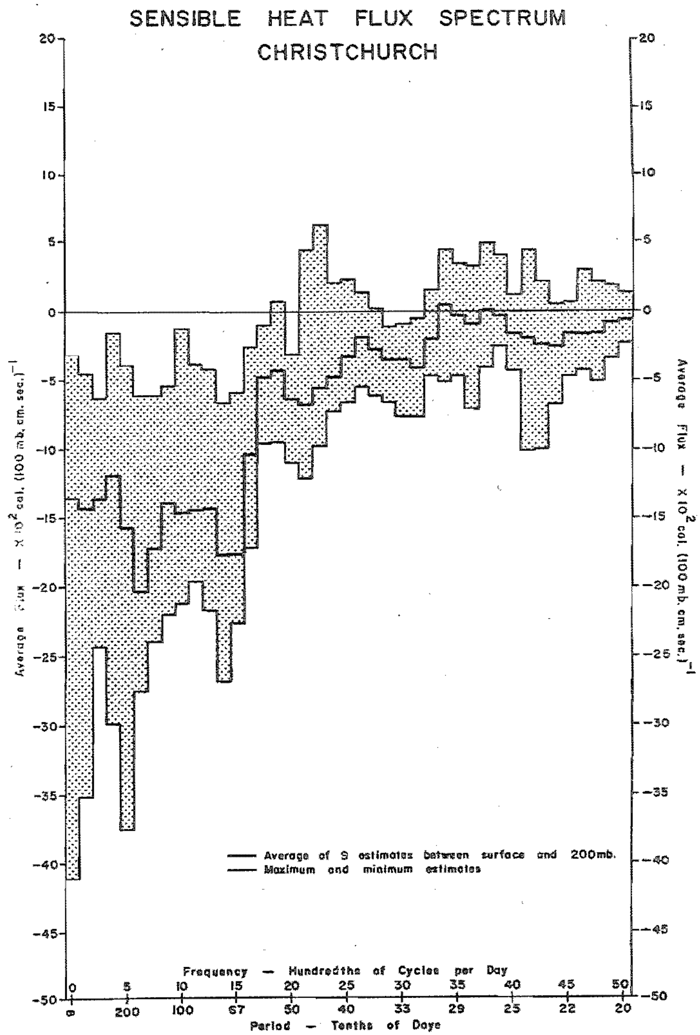


Fig. 6.4.2.4

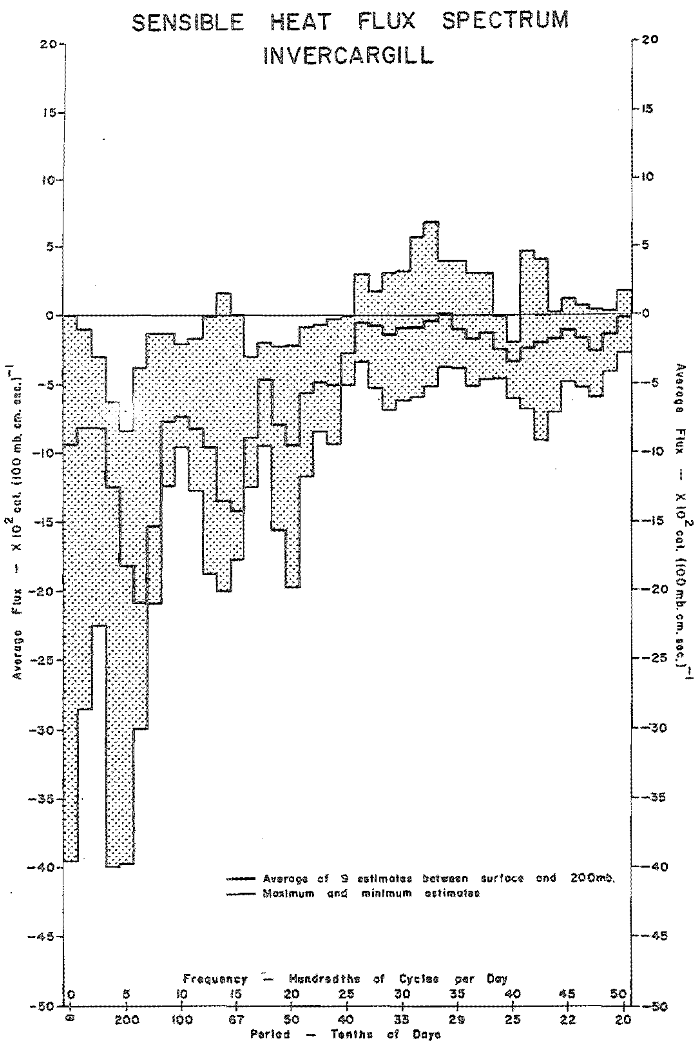


Fig. 6.4.2.5

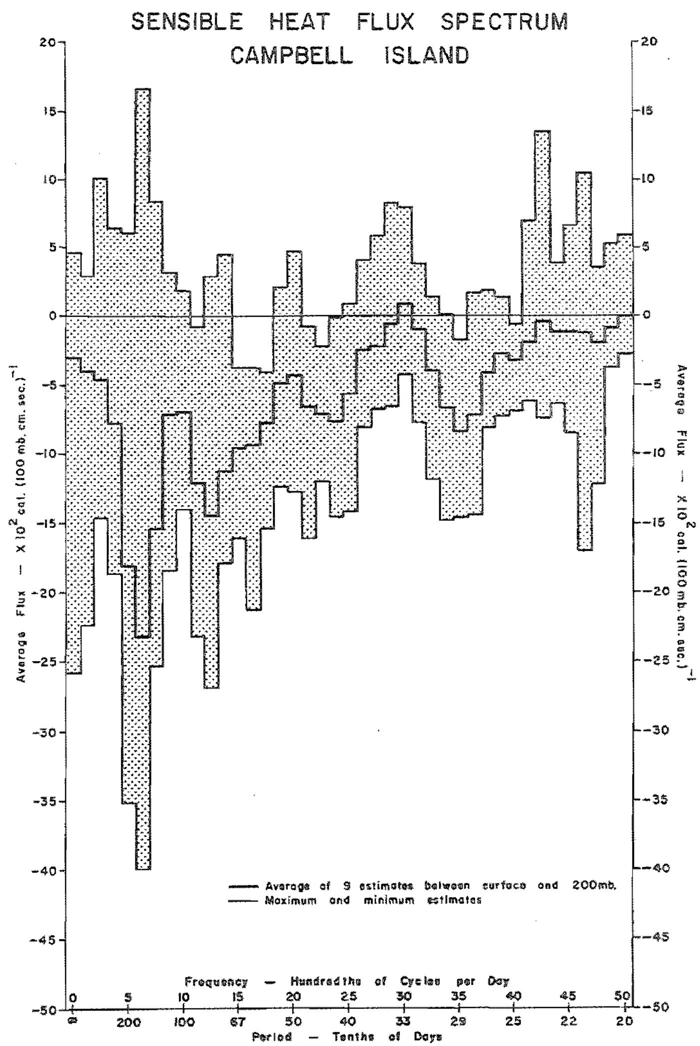


Fig. 6.4.2.6

6.4.3 Water Vapour

The total flow of water vapour is clearly depicted in Fig. 6.4.3.1. All stations reveal southward transports which reach maxima in the 900 - 800 mb layer. The decrease towards the surface is due to reduced wind speed through friction, and the decrease above to decreasing water vapour content. Maximum values occur at Campbell Island owing to strong wind fluctuations. The relatively low values at Christchurch and Invercargill are associated with their locations with respect to the mountain ranges.

The annual transports for water vapour are similar to those of sensible heat (Table 6.4.3.1).

The frequency distributions of transfer show (Figs. 6.4.3.2 to 6.4.3.6) marked movement towards longer periods as latitude decreases. There is less consistency than in temperature but peaks still appear at about 0.0250, 0.0625, 0.1375 and 0.2000 cycles per day. The high level spectra for Campbell Island are again erratic and create a negative peak in the high frequencies of the average spectrum. Much of the power here is probably real but some will be produced by aliasing.

Flux of water vapour by the mean wind is of the same order of magnitude as the eddy component.

Latent heat transfer is an important element in the total heat budget for the lower troposphere, and calculations from this section may be converted to heat units by the application of the latent heat of evaporation.

TABLE 6.4.3.1

WATER VAPOUR TRANSFER

gm (100 mb cm sec)$^{-1}$

	Transient Eddy $\overline{V'q'}$	Annual Eddy Component	Total Eddy	Mean Component
Nandi				
Surface	- 20.1	- 4.9	- 25.0	- 22.6
900 mb	- 16.8	- 7.4	- 24.2	- 98.3
800	- 35.0	- 2.7	- 37.7	- 132.0
700	- 30.9	- 0.1	- 31.0	- 87.8
600	- 28.0	+ 1.7	- 26.3	- 61.3
500	- 18.1	+ 2.6	- 15.5	- 37.6
400	- 13.2	+ 1.6	- 11.8	- 12.0
300	- 3.8	+ 0.3	- 3.5	- 0.3
Whenuapai				
Surface	- 32.0	+ 0.5	- 31.5	- 7.4
900 mb	- 39.5	+ 4.0	- 35.5	- 36.4
800	- 48.3	+ 1.2	- 47.1	- 4.0
700	- 39.0	+ 2.2	- 36.8	+ 3.0
600	- 24.0	+ 1.6	- 22.4	- 2.2
500	- 12.0	+ 1.8	- 10.2	- 1.9
400	- 7.0	+ 1.2	- 5.8	- 2.9
Christchurch				
Surface	- 5.9	- 2.1	- 8.0	+ 10.7
900 mb	- 1.2	- 3.7	- 4.9	- 35.8
800	- 22.4	- 2.7	- 25.1	- 14.1
700	- 24.7	- 1.2	- 25.9	+ 3.7
600	- 21.3	0.0	- 21.3	+ 7.2
500	- 8.4	- 0.4	- 8.8	+ 1.8
400	- 5.9	- 0.3	- 6.2	- 1.5
Invercargill				
Surface	- 4.9	+ 5.7	+ 0.8	+ 42.5
900 mb	- 9.3	- 2.3	- 11.6	- 70.5
800	- 22.6	- 2.9	- 25.5	- 57.8
700	- 25.1	- 0.9	- 26.0	+ 17.8
600	- 18.5	- 1.3	- 19.8	- 12.4
500	- 11.6	- 1.4	- 13.0	- 8.5
400	- 5.3	- 0.5	- 5.8	- 5.6
Campbell Is.				
Surface	- 30.9	- 5.1	- 36.0	- 34.7
900 mb	- 61.1	- 5.7	- 66.8	- 87.0
800	- 43.7	- 3.2	- 46.9	- 34.6
700	- 14.8	- 3.4	- 18.2	- 18.4
600	- 7.3	- 1.1	- 8.4	- 8.3
500	- 0.4	- 0.2	- 0.6	- 4.3

EDDY TRANSFER OF WATER VAPOUR
AUGUST 1962–AUGUST 1963

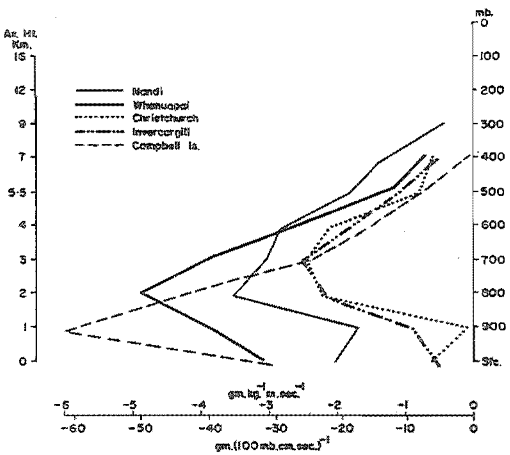


Fig. 6.4.3.1

VAPOUR FLUX SPECTRUM
NANDI

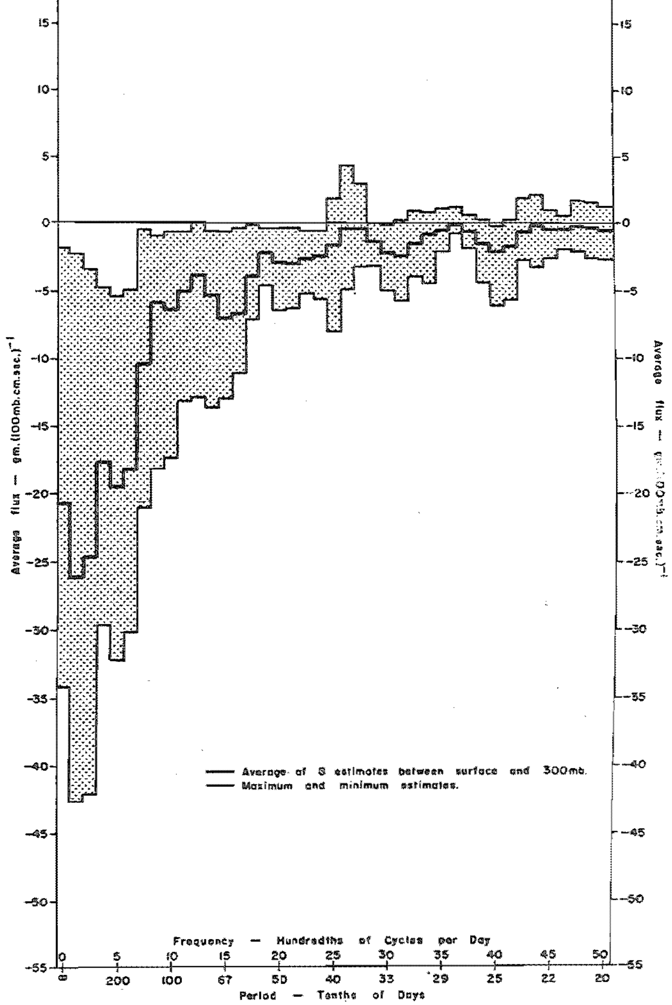


Fig. 6.4.3.2

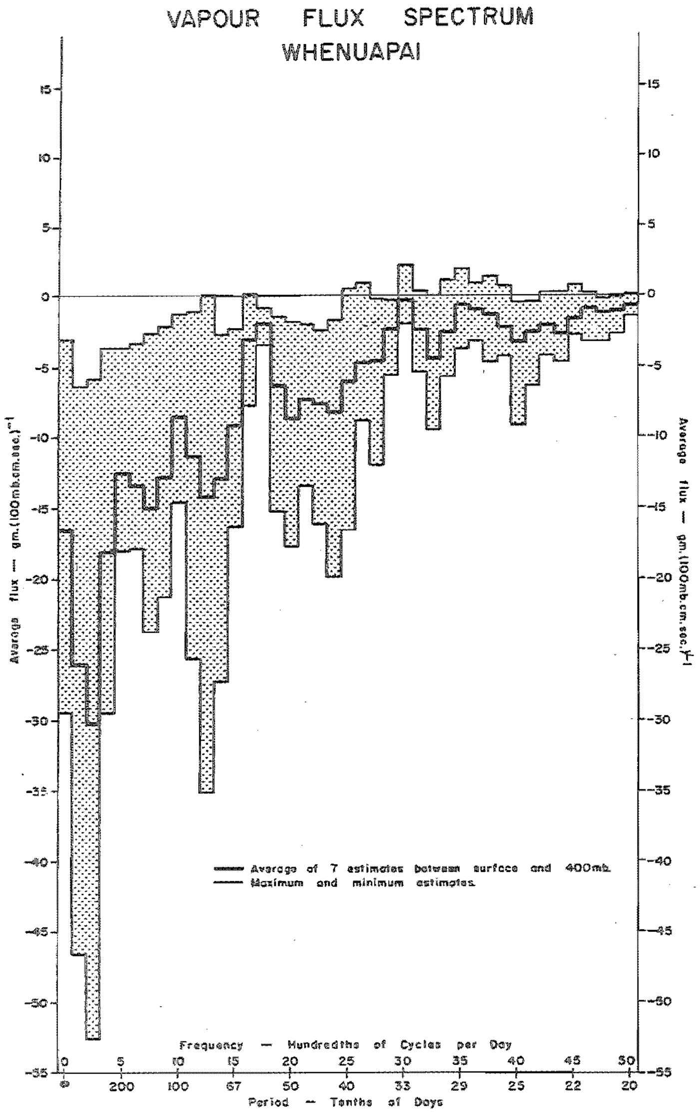


Fig. 6.4.3.3

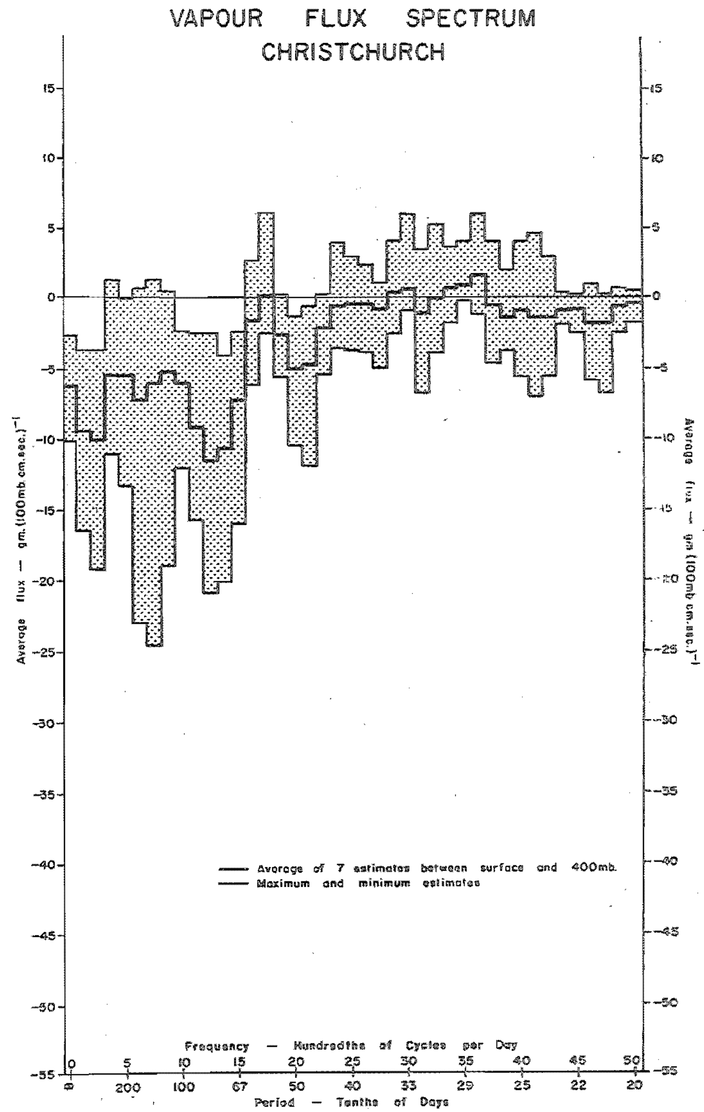


Fig. 6.4.3.4

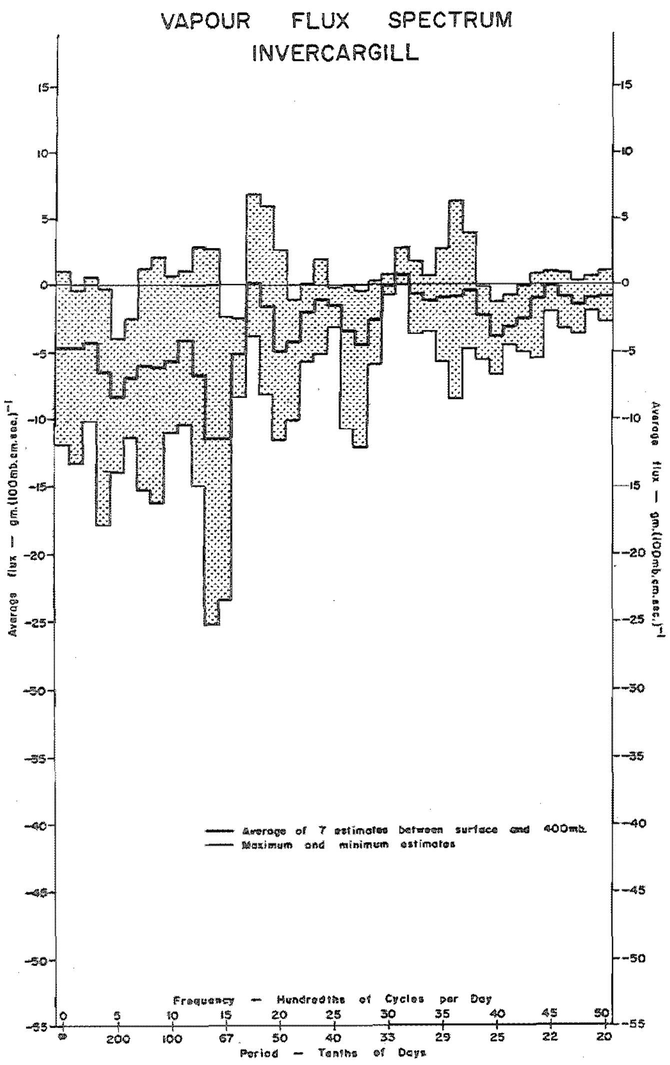


Fig. 6.4.3.5

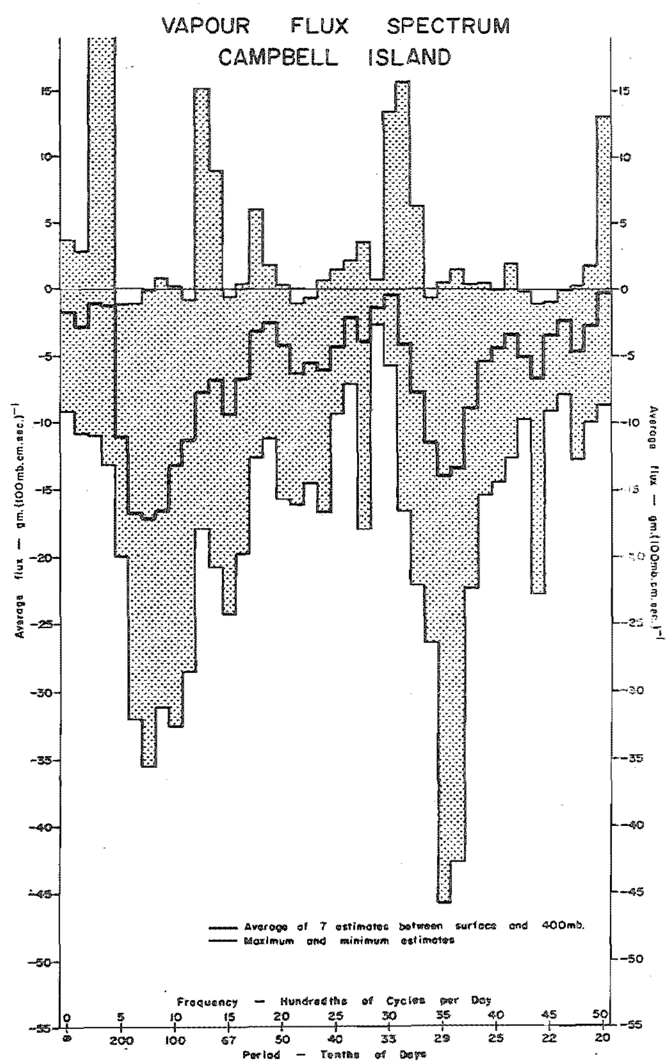


Fig. 6.4.3.6

6.4.4 Summary of Transfers

It is clear from the calculations that the models constructed in Chapter 5 are effective in transferring both sensible and latent heat southwards, and that they are therefore important agents in the conversion of zonal available potential energy into eddy available potential energy. In terms of scale of transport the lower frequencies are obviously the important ones. This is in agreement with other investigations which show that this conversion is due to the long slowly moving wave disturbances.

It has also been inferred here from the precipitation data that these same frequencies convert eddy available potential energy into eddy kinetic energy. The same calculations show that the latent heat released during such a process, although small, serves to generate or intensify eddy available potential energy.

Relative angular momentum transfer is irregular and its direction in the energy conversion process is uncertain. This is true only to a limited extent of the spatial domain according to Wiin-Nielsen et al. (1964) but the average exchange is always directed towards the zonal flow (Saltzman et al. 1964).

In many of the transports the annual oscillation makes a significant contribution to the total eddy flow. This is particularly true of relative angular momentum fluxes at Nandi and Whenuapai (Table 6.4.1).

Since the mean meridional wind is from the north at all stations and levels except for Auckland and

Christchurch in the mid-troposphere, the standing waves and meridional terms are directed towards the south. This, according to the New Zealand Meteorological Service (1962), is the normal arrangement. As it is unlikely that the mean meridional circulation is very strong, most of the northerly wind must be due to a stationary wave pattern. Lamb (1959) has shown that such a pattern exists in the Southern Hemisphere, probably as the result of the Andean Chain, with one ridge centred just to the east of New Zealand. It should be noted that whilst the stationary waves are masked by the mean terms of point time series analysis they are incorporated into, and may dominate, the spatial scale estimates.

6.5 Climate

It is frequently acknowledged that climate is "the synthesis of weather" (Durst 1951) but seldom has "synthesis" been accomplished. Early in the present work it was argued that the arithmetic mean was not sufficient for the description of climate, let alone explanation, and that the incorporation of atmospheric perturbations, the features of process, was essential. These disturbances appear to be unavoidable consequences of the unequal heating of the atmosphere and are necessary for the general circulation of the atmosphere. Furthermore, the means must be recognised as more than just statistical averages. Rather, they are the balance points between all associated processes: the points at which quasi-equilibrium is maintained.

An explanation of climate then requires "a clear insight into the dynamical mechanisms of the general circulation" (Pfeffer 1960). Some of the broad features of this system were outlined in section 6.2 of this chapter with some local elements calculated for New Zealand in section 6.4 It is seen that in the final analysis the whole atmosphere, not just one region or one hemisphere, must be used in explanation. However, the local climate may be described by the observations recorded therein and the components may be assigned to the various processes in the circulation. It is their presence and intensity in any one particular area which can be explained only through an investigation into the whole circulation.

The link between the components and the circulation in one region has been made with the statistical time scale models of chapter 5. They are strictly neither dynamic nor synoptic models as defined by Godske et al. (1957) but they bridge the gap between the instantaneous spatial patterns of meteorology and the times series of point climate.

This is only one approach to a highly complex problem which will be solved only through the acquisition of more empirical data and their merging with more advanced dynamical theory.

CHAPTER 7

SUMMARY AND CONCLUSIONS

At the outset dynamic climatology was defined as the explanatory description of climate in terms of the circulation or disturbances of the atmosphere. Its application to the New Zealand region posed an initial problem of method since there is a lack of meteorological data in the Southern Hemisphere and there is no recognised technique of climatic explanatory description.

Whilst the dynamic processes of weather, the components of climate, are necessarily studied in the instantaneous time domain by differential equations, there is, as yet, no way of integrating over very long periods to produce meaningful results. This study therefore began with a new approach; that of isolating the frequency components of climatic time series and interrelationships. The method of spectral analysis was found to be a powerful tool although a number of statistical problems existed. It was shown that the difficulties of handling large quantities of data and aliasing can be overcome by smoothing and decimation but the most serious problem of non-stationarity remains. Its presence does, to some extent, limit the application of this technique although its influence may be gauged and reduced by successive calculations of the same spectra for data from different periods.

Basically, the result of spectral analysis is to average the variability of a series or the interrelationships of two series into groups of specific frequencies. It has the desired effect of producing "average", in time rather than in space, disturbances which have meaning in terms of the general circulation and which help both to describe and to explain the broad features of climate.

It should be realized that this is only one of several possible approaches to the study of climate and that the investigation has been limited to a specific size range of phenomena. The range chosen allows a discussion only of frequencies between 0.0125 and 0.0400 cycles per day (80 to 2½ day period) together with the annual and diurnal cycles. These frequencies appear to be the important ones for the general circulation and regional climate but much variability remains unaccounted for in the interrelationships and in the unresolved higher frequencies. Also, the seasonal variation in scale has not been studied although spatial Fourier analysis reveals marked differences between winter and summer.

In summary the following conclusions may be drawn from this study:

(i) Sinusoidal curves, fitted by least squares, may be used successfully in the initial removal and description of trend components, whilst classical Fourier analysis is found to be useful in the more detailed description of the normals of temperature and precipitation. However, when applied to frequencies other than the diurnal and annual cycles their physical meaning is difficult to interpret.

(ii) Spectral and cross spectral analyses of meteorological time series are most useful tools in the description of large scale mid-latitude climates.

(iii) All variables studied show continuous spectra with concentrations mainly in the lower frequencies. The individual peaks (except for the diurnal and annual cycles) are not usually statistically significant.

(iv) Despite (iii), certain frequencies of about 0.0250, 0.0625, 0.1500 and 0.2000 cycles per day consistently stand out in most spectra which suggests that these were the physically significant frequencies in the year under study.

(v) Models, constructed from the variance spectra and the spectra of the interrelationships, provide the link between the fluctuations in climatic time series and the general circulation. They show remarkable resemblance to the instantaneous spatial systems despite the averaging of waves of different lengths and speeds.

(vi) The low to medium frequency disturbances (< 0.2 cycles per day) are shown to be the important ones in the eddy transport of sensible and latent heat southwards and therefore in the conversion of zonal available potential energy into eddy available potential energy. However, there is also a definite shift to the lower frequencies at lower latitudes especially with moisture.

(vii) The calculated eddy transfer of relative angular momentum is found to be much smaller than expected. It suggests that disturbances in the region of New Zealand during the period were more symmetrical than usual.

(viii) Transports by the standing waves and mean meridional components are directed towards the south over New Zealand.

The aims of this investigation of producing and of testing a new method of climatic description and explanation have in large measure been achieved. The study has demonstrated that the large-scale features of climate may be objectively related to the general circulation of the atmosphere even in areas where data are limited. Furthermore, it has made it clear that future advances in dynamic climatology must occur through the full integration of the spatial and time fields. As an initial step the present technique may be profitably applied to other areas having more detailed data and to other time scales.

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APPENDIX

LIST OF MATHEMATICAL SYMBOLS AND CONSTANTS

$a_{j,t}$	trend functions e.g. $\cos\theta_t$ (for mean $a_{j,t} \equiv 1$)
A , subscript	available potential energy
A_j	amplitude, coefficient of Fourier series
AP	available potential energy
$\mathbb{C}$	translation vector
c_p	specific heat at constant pressure, 0.2400 ITcal gm^{-1} $^{o}K^{-1}$
$C_{(r)}$	cospectrum
CN	conversion process
d	integer referring to decimation
d , subscript	see (1) F_d (2) R_d (3) γ_d
D_F	Frictional dissipation
D_h	difference of average positive and negative lagged products
e , subscript	eddy component
f	an atmospheric quantity
$f_{(r)}$	frequency (= $\frac{r}{2m\Delta t}$)
F_d	decimation, retention of every d^{th} item
g	acceleration of gravity, 980 cm sec^{-2}
G_h	average lagged products
GE	generation of potential energy
h , also as subscript	lag
H	diabatic heat
j , also as subscript	integer referring to a specific trend function
k , subscript	kinetic energy
$K_{(r)}$	cross variance (half cross amplitude)

KE	kinetic energy
$L_{(r)}$	line power
m, also as subscript	maximum lag
M_n	average over n values
n, also as subscript	number of values over which average taken
N	total number of observations
p	pressure
$P_{(r)}$	population value of the variance of $S_{(r)}$
$PE+IE$	total potential and internal energy
q, also as subscript	specific humidity approximated by the mixing ratio
$Q_{(r)}$	quadrature spectrum
r, also as subscript	integer referring to frequency (see $f_{(r)}$)
$R_{(r)}$	frequency response
R_d	gas constant for dry air, 2.8705×10^6 erg gm^{-1} $^oK^{-1}$
s	an atmospheric quantity
$S_{(r)}$	variance spectrum
$S'_{(r)}$	unadjusted variance spectrum
t, also as subscript	time
T, also as subscript	temperature
$\overline{T}_v$	mean virtual temperature with respect to height
u, also as subscript	W-E component of wind
v, also as subscript	S-N component of wind
$\mathbb{V}$	wind vector

w	vertical component of wind
$W_{(r)}$	coherence
x	distance W-E direction
x, subscript	x time series
x_t	time series with zero mean, trend removed; x_t', x_t'', smoothed and decimated values (see Fig. 2.6.1)
X_h	lagged cross product
y	distance S-N direction
y, subscript	y time series
y_t	time series with zero mean, trend removed
z	zenith distance
z, subscript	zonal component
z_t	unmodified time series
z_t^+	monthly normal values of time series
$\hat{\beta}_j$	estimated coefficients of $\alpha_{j,t}$
γ	lapse rate, $\frac{\partial T}{\partial z}$
γ_d	dry adiabatic lapse rate $(= g/c_p)$, 9.753°C km^{-1}
Δt	time interval between observations
Δz	thickness, height difference between pressure surfaces
θ	time angle
ν	degrees of freedom
π	3.14159265
$\tau_{(r)}$	lag between series
Φ	phase angle
χ^2	chi-square
∇	del operator
∇_H	horizontal components of del operator